**ABSTRACTS OF
IWOTA 2008**

IWOTA 2008

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Index

Exact solutions to the nonlinear Schrödinger equation

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A systematic method is presented to construct exact solutions to the nonlinear Schrödinger equation on the line. An explicit formula and its equivalents are provided to express such exact solutions in a compact form using matrix exponentials. Such exact solutions can alternatively be written explicitly as algebraic combinations of exponential, trigonometric, and polynomial functions of the spatial and temporal coordinates. The method is generalizable to some other nonlinear partial differential equations such as the Korteweg-de Vries equation on the half line and the sine-Gordon equation by exploiting the separability of the kernel of the associated Marchenko integral equation. This is based on the joint work with C. van der Mee and F. Demontis of University of Cagliari, Italy.

Weighted composition operators on the Bloch space on bounded homogeneous domains

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Let $\mathbb{D}$ denote the unit disk in $\mathbb{C}$. For $\psi : \mathbb{D} \to \mathbb{C}$ and $\varphi : \mathbb{D} \to \mathbb{D}$ the weighted composition operator is define by $W_{\psi,\varphi}(f) = \psi(f \circ \varphi)$. In their recent work, Ohno and Zhao characterized the bounded and compact weighted composition operators on the Bloch space and little Bloch space of the unit disk. In this talk, we will explore to what extent these results can be extended to the Bloch space on bounded homogeneous domains in $\mathbb{C}^n$, and when necessary, to the case of the unit ball $\mathbb{B}_n$. As special cases, we discuss multiplication and composition operators on the Bloch space on bounded homogeneous domains. This is based on joint work with F. Colonna.

Strichartz estimate for the wave equations on a deformation of the Heisenberg group

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We study **Strichartz estimate** for the wave equations on a deformation of the Heisenberg group. We give a brief description of the ideas, constructions, results, and prospects of this theory.

Moment problems for real measures on the unit circle

Mihály Bakonyi

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In this talk we are considering the following problem: when are the given complex numbers $(c_j)_{j=-n}^n$, $c_{-j} = \bar{c}_j$, the first moments of a real Borel measure $\mu = \mu^+ - \mu^-$ on $\mathbb{T}$, such that μ^- is supported on a set of at most k points. A necessary and sufficient condition is that the Toeplitz matrix $T = (c_{i-j})_{i,j=0}^n$ is a certain real linear combination of rank 1 Toeplitz matrices. For $k > 0$, this is more general than the condition that T admits Hermitian Toeplitz extensions with k negative squares. For a singular T, an equivalent condition is that a certain polynomial has all its roots on $\mathbb{T}$. We also discuss the situation when T is invertible.

This talk is based on joint work with Ekaterina Lopushanskaya.

Robust H-infinity control theory and Nevanlinna-Pick interpolation: extensions to multidimensional systems and multivariable functions

Joseph Ball

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The connection between the standard problem of H-infinity (or robust) control and Nevanlinna-Pick interpolation was one of the main influences leading to the development of H-infinity control theory on the one hand and extensions of Nevanlinna-Pick interpolation to more general matrix- and operator-valued settings on the other. Since then there have been a number of extensions of Nevanlinna-Pick-type interpolation theory to multivariable settings, and, independently, extensions of the robust control theory to various sorts of multidimensional linear systems on the other. In particular the state-space solution of the suboptimal H-infinity control problem from the 1990s based on strict Linear Matrix Inequalities rather than on Riccati equations has straightforward generalizations to the context of various types of multidimensional systems and can be seen as leading to solutions of various sorts of multivariable Nevanlinna-Pick interpolation problems, but without the linear-fractional parametrization for the solution set. In the other direction, recent developments in multivariable interpolation theory may be of interest for control. This talk will survey these developments and connections.

The state space approach to factorization

Harm Bart

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The talk is concerned with the state space method for factorization of matrix and operator functions as presented in the recently published work by Bart, Gohberg, Kaashoek, Ran: Factorization of Matrix and Operator Functions: The State Space Method, OT 178, Birkhäuser, 2008. Special attention will be given to possibly non-minimal factorization of matrix functions into factors of McMillan degree one. Such factorizations are related to the two machine flow shop problem from the theory of combinatorial job scheduling. Taking advantage of this connection examples are obtained where the factorization in question can be computed explicitly. Stability matters are discussed too.

Toeplitz operators and random matrices

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This talk is intended to be a survey of the connections between the theory of Toeplitz operators and computations of quantities that arise in random matrix theory. For example, certain density functions in random matrix theory can be described using Toeplitz determinants. We will illustrate how this is done and then show how random matrix theory can predict certain results about operators. If time permits, we will also discuss some new results for a class of perturbed random matrices.

A C^*-algebra of operators with shifts having a nonempty set of periodic points

M.Amélia Bastos

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An invertibility theory in a non local C^*-algebra generated by multiplication operators by piecewise slowly oscillating functions and by a group of unitary operators associated to an amenable group of orientation preserving homeomorphisms on the unit circle having a nonempty set of periodic points, is presented. The invertibility theory is obtained by combining local trajectory methods for C^*-algebras associated with C^*-dynamical systems, with a redution of functional operators with shifts having periodic points to matrix functional operatos with shifts having fixed points.

This is joint work with C.A. Fernandes and Y.I. Karlovich.

Fast algorithms for polynomial–Vandermonde matrices related to quasiseparable matrices

Tom Bella
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The interplay between polynomials and dense structured matrices is a classical topic. Structure in this sense is interpreted to mean their n^2 entries can be "compressed" to a smaller number $\mathcal{O}(n)$ of parameters. Operating directly on these parameters allows one to design efficient *fast algorithms* for these matrices and for the related applied problems.

In the past decades matrices with structures such as DFT/DCT/DST, Toeplitz, Hankel, Vandermonde or Cauchy structure were the focus of attention. In this talk, some results that demonstrate that a relatively new *quasiseparable* structure enables substantial generalizations of a number of different algorithms will be presented.

Some results to be presented are joint work with Israel Koltracht, to whom the minisymposium is dedicated.

Inverse Stieltjes like functions and systems with Schrödinger operator

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We consider rigged canonical systems of the form

$$\Theta = \begin{pmatrix} \mathsf{A} & K & 1 \\ \mathcal{H}_+ \subset L_2[a,+\infty) \subset \mathcal{H}_- & & \mathbf{C} \end{pmatrix}, \tag{1}$$

whose main operator A is based on a Schrödinger operator T_h in $L_2[a,+\infty)$ with a non-selfadjoint boundary condition

$$\left\{ \begin{array}{l} T_h y = -y'' + q(x)y \\ y'(a) = hy(a) \end{array} \right. , \quad \left(q(x) = \overline{q(x)},\ \mathrm{Im}\ h \neq 0 \right). \tag{2}$$

A class of scalar inverse Stieltjes like functions is realized as linear-fractional transformations of transfer mappings of systems (1). In particular it is shown that any inverse Stieltjes function of this class can be realized in the unique way so that A in (1) possesses a special semi-boundedness property. We derive formulas that restore the system uniquely and allow to find the exact value of a non-real parameter h in (2) as well as a real boundary parameter μ that appears in the definition of A. An elaborate investigation of these formulas shows the dynamics of the restored parameters h and μ in terms of the changing free term α from the integral representation of a given realizable function. These results are based on a joint work with Eduard Tsekanovskii and were recently submitted for publication [1].

References

[1] S.V. Belyi and E.R. Tsekanovskii. *Inverse Stieltjes like functions and inverse problems for systems with Schrödinger operator*, Integr. Equ. Oper. Theory (submitted).

Completely positive kernels

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We shall review some of the recent results on completely positive kernels taking values in the space of bounded operators between two C* - algebras and see to what extent and to what advantage, the complete positivity can be dropped.

A comparison of spectra for some nonlinear eigenvalue problems

Paul Binding
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A review will be given of some recent work on nonlinear versions of Sturm-Liouville theory, including Berestycki's half-eigenvalues, Fučík's spectrum and the p-Laplacian, for various boundary conditions. Some of the results are quite similar to the classical ones, but some are markedly different.

Multivariable generalization of the Schur class and some weighted interpolation problems

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The operator-valued Schur-class is defined to be the set of holomorphic functions mapping the unit disk into the space of contraction operators between two Hilbert spaces. Various multivariable generalizations of this class have appeared recently, one of the most encompassing being that of Muhly and Solel where the unit disk is replaced by the strict unit ball of the elements of a dual correspondence E^σ associated with a W^*-correspondence E over a W^*-algebra $\mathcal{A}$ together with a $*$-representation σ of $\mathcal{A}$. We discuss the notion of reproducing kernel Hilbert correspondence in this setting and identify the Muhly-Solel Hardy spaces as reproducing kernel Hilbert correspondences associated with a completely positive analogue of the classical Szegö kernel. We also discuss a certain lifting theorem in this setting which generalizes the Commutant Lifting Theorem by Muhly-Solel in this setup and give some applications to "weighted" interpolation problems. This is a joint work with J. Ball, Q. Fang and S. ter Horst

Probability in matrix theory

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Solving large systems $Ax = y$ of linear equations is problematic if the norm of A^{-1} is unknown or very large. The purpose of the talk is to illustrate that if worst case analysis is replaced by probability arguments, then useful conclusions are possible without knowing the norm of A^{-1} and the equation may be well-posed even if the norm of A^{-1} is available but very large. We also demonstrate that certain inequalities, for instance estimates for the commutator of two matrices or so-called Marcinkiewicz-Zygmund inequalities for sample values of spherical harmonics, can be spectacularly improved if one is satisfied by having these inequalities with a given probability only. The talk is based on joint work with S. Grudsky, D. Wenzel, D. Potts, and S. Kunis.

Interpolation problems for Schur-Agler functions of the unit ball: from Nevanlinna-Pick to Abstract Interpolation Problem

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An operator valued function S analytic on the unit ball $\mathbb{B}^d$ of $\mathbb{C}^d$ is called a Schur-Agler function if the kernel $\frac{I-S(z)S(\zeta)^*}{1-\langle z,\zeta\rangle}$ is positive on $\mathbb{B}^d \times \mathbb{B}^d$. In the talk, we will survey various increasingly more general operator-theoretic formulations of generalized Nevanlinna-Pick interpolation for Schur-Agler functions: Nevanlinna-Pick problem – Commutant Lifting/Sarason problem – Operator Argument interpolation problem – Abstract interpolation problem. Chain-matrix and Redheffer-type linear-fractional parametrizations for the set of all solutions will be discussed.

The talk is based on a joint work with J. A. Ball.

Inertia of certain structured Hermitian matrices

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In the talk, we will discuss two classes of structured Hermitian matrices are considered with the additional property that certain principal submatrices are all singular. Such matrices can be considered as the Pick matrices of certain (interior and boundary) norm constrained interpolation problems for functions meromorphic on the unit disk which the iterative Schur algorithm does not apply to. We characterize these matrices in terms of the parameters determining their structure and present formulas for their inertia.

Computing eigenvalues by Poincare determinants

Amin Boumenir

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We show how the Poincare determinant can be used to compute eigenvalues of singular Sturm-Liouville operators. Lidskii's theorem and the finite section method offer simple solutions to difficult issues in computational spectral theory such as reducing numerical integration. This framework provides a direct way to compute these eigenvalues for operators of trace and Hilbert-Schmidt classes at low cost. Estimates on the truncation error of the finite section and practical ways of filling up infinite matrices are provided by the same approach. This talk is based on joint work with Vu Kim Tuan.

Adjoint composition operators on $H^2(\mathbb{U})$ induced by strongly outer regular rational selfmaps of $\mathbb{U}$

Paul Bourdon

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We give a sufficient condition "strong outer regularity" for a rational self-map of the open unit disk $\mathbb{U}$ to induce a composition operator on $H^2(\mathbb{U})$ whose adjoint is a rank-one perturbation of a sum of weighted composition operators. We also show that if a strongly outer regular rational selfmap φ has the property that $\phi(\partial\mathbb{U}) \cap \partial\mathbb{U}$ is a singleton, then C_φ^* is a compact perturbation of a weighted composition operator. (Joint work with Joel H. Shapiro)

Differential forms in Hermitean Clifford analysis

Fred Brackx

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Euclidean Clifford analysis is nowadays a well established branch of classical analysis centred around the notion of monogenic functions, in particular null solutions of the rotation invariant Dirac operator. Recently so–called Hermitean Clifford analysis emerged as a refinement of Euclidean Clifford analysis. Hermitean Clifford analysis is based on the introduction of an additional datum, a so–called complex structure, in order to bring the notion of monogenicity closer to complex analysis. The complex structure J induces an associated, called twisted, Dirac operator ∂_J. Hermitean Clifford analysis then focuses on Hermitean monogenic functions, i.e. simultaneous null solutions of both operators ∂ and ∂_J, in this way breaking down the rotational invariance of the Dirac operator, reducing it to $\mathrm{U}(n)$-symmetry for the considered Hermitean Dirac system.

In a rather formal way differential operators occurring in Hermitean Clifford analysis may be associtated with similar differential forms appearing in complex analysis on $\mathbb{C}^n$. It turns out that the differential forms obtained are the well-known forms of analysis on Kählerian manifolds. However in this talk we restrict ourselves to the flat Kähler metric on $\mathbb{C}^n$ with fundamental form $\Omega = \frac{i}{2}\, \partial\overline{\partial}|z|^2$. An account is given of all interesting differential operators and forms. By exploiting the transition scheme between operators and forms, new relations between the differential operators considered are established.

Defect eigenvalues and Diophantine conditions

Jared Bronski

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We consider the problem of a Schrodinger operator with a potential given by a periodic potential plus a compactly supported “defect”. We derive an index which counts the number of point eigenvalues in a given gap of the periodic spectrum. We also derive the following large gap number asymptotics: Every sufficiently large numbered gap contains exactly one defect UNLESS a certain Diophantine approximation problem has solutions. In this case there exists a subsequence of gaps (related to the solutions of the Diophantine problem) which may get zero, one or two defect eigenvalues. (This is joint work with Zoi Rapti (Illinois)).

Spectral properties of a class of q-difference operators

Brian Brown

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We study the spectral properties of a class of Sturm-Liouville type operators on the real line where the derivatives are replaced by a q-difference operator which has been introduced in the context of orthogonal polynomials. Using the relation of this operator to a direct integral of doubly-infinite Jacobi matrices, we construct examples for isolated pure point, dense pure point, purely absolutely continuous and purely singular continuous spectrum. It is also shown that the last two spectral types are generic for analytic coefficients and for a class of positive, uniformly continuous coefficients, respectively.

On extension of locally defined indefinite functions on ordered groups

Ramon Bruzual

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A definition of k-indefinite function of archimedean type, on an interval of an ordered group Ω with an archimedean point is given. It is said that Ω has the indefinite extension property if every continuous k-indefinite function of archimedean type, on an interval of Ω, can be extended to a continuous k-indefinite function on the whole group Ω.

It is shown that if a group Γ is semi-archimedean and it has the indefinite extension property, then $\Gamma \times \mathbb{Z}$ with the lexicographic order and the product topology has the indefinite extension property. As a corollary it is obtained that the groups $\mathbb{Z}^n$ and $\mathbb{R} \times \mathbb{Z}^n$, with the lexicographic order and the usual topologies, have the indefinite extension property. This is a joint work with Marisela Domínguez

References

[1] R. Bruzual and M. Domínguez, *Extension of locally defined indefinite functions on ordered groups*, Integral Equations Operator Theory, **50** (2004) 57-81.

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[3] M. Grossman and H. Langer, *Uber indexerhaltende Erweiterungen eines hermiteschen operators in Pontrjaginraum.* Math. Nachrichten **64** (1974), 289-317.

[4] M.G. Kreĭn and H. Langer, *On some continuation problems which are closely related to the theory of operators in spaces Π_κ IV. Continuous analogues of orthogonal polynomials on the unit circle with respect to an indefinite weight and related continuation problems for some classes of functions.* J. Operator Theory **13** (1985), 299-417.

Asymptotic analysis of the semiclassical sine Gordon equation

Robert Buckingham

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The small dispersion sine-Gordon equation models magnetic flux propagation in long Josephson junctions. We consider a family of Satsuma-Yajima type initial data for which the scattering data has recently been computed. Plots of exact solutions obtained by the inverse scattering method for small dispersion reveal regions of pure librational and rotational motion, as well as regions of multi-phase waves separated by primary and secondary nonlinear caustics. We use the Riemann-Hilbert approach to study the leading asymptotic solution in the zero dispersion limit in certain regions and consider the transitions between regions. This is joint work with Peter Miller.

Boundedness and stability for evolution families of operators acting on Banach spaces

Constantin Buşe

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A result of E. A. Barbashin states that an exponentially bounded evolution family $\{U(t,s)\}_{t\geq s\geq 0}$ defined on a Banach space satisfying some measurability conditions is uniformly exponentially stable if and only if for some $1 \leq p < \infty$, the inequality $\sup_{t\geq 0}\int_0^t ||U(t,s)||^p ds < \infty$ is fulfilled. Actually the Barbashin result was formulated for non-autonomous differential equations in the

framework of the finite dimensional spaces. We prove that the above "uniform" condition can be replaced by a "strong" one along the trajectories of the dual family.

Among others it is shown that the evolution family $\{U(t,s)\}$ is uniformly exponentially stable if there exists a non-decreasing function $\phi : \mathbf{R}_+ \to \mathbf{R}_+$ with $\phi(r) > 0$ for all $r > 0$ such that for each $x^* \in X^*$, the inequality $\sup_{t\geq 0} \int_0^t \phi(||U(t,s)^* x^*||) ds < \infty$ is fulfilled. Related results for periodic evolution families are also obtained. This talk is based on a joint work with A. D. R. Choudary, S. S. Dragomir and M. S. Prajea.

Matrix Wiener-Hopf factorization and Riemann-Hilbert problems in a Riemann surface

Cristina Câmara

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The central topic of this talk is the interplay between the problem of Wiener-Hopf factorization of 2x2 matrix functions with Holder continuous elements and scalar boundary value problems on Riemann surfaces. It is shown that an appropriate characterization of classes of 2x2 matrices of that type enables us to associate a Riemann surface S with each class and to reduce the problem of Wiener-Hopf factorization to solving a scalar Riemann-Hilbert problem on S. For the solution of this problem, a notion of S-factorization is introduced and discussed.

Composition operators with maximal norm on weighted Bergman spaces

Brent Carswell

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The composition operators with maximal norm acting on the classical Hardy space are those which are induced by inner functions. In this talk, we discuss the problem of determining which composition operators have maximal norm when acting on the standard radially weighted Bergman spaces. This is joint work with C. Hammond.

The Kadison-Singer problem in mathematics and engineering

Peter Casazza

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We will see that the famous intractable 1959 Kadison-Singer Problem in C^*-algebras is equivalent to fundamental unsolved problems in a dozen areas of research in pure mathematics, applied mathematics and engineering. This gives all these areas common ground on which to interact as well as explaining why each area has volumes of literature on their respective problem without a satisfactory resolution.

Clifford analysis and reflection invariant Dirac operators

Paula Cerejeiras

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In the 80's Dunkl initiated the study of differential-difference operators associated to specific finite reflection groups. These operators - often called Dunkl operators - possess the property of invariance under the associated group. In this talk we present the basic ingredients for a Clifford analysis of reflection invariant Dirac operators, such as a Fischer decomposition and an explicit construction of a Cauchy kernel.

Fast algorithms for minimum Sobolev norm methods

Shivkumar Chandrasekaran

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We present a new paradigm, Minimum Sobolev Norm (MSN) methods, for discretizing numerical problems, that has some potential advantages over standard methods. For example, we show that when applied to the classical function interpolation problem, it can overcome the Runge phenomenon. The discrete problems that arise via this scheme can be dense matrices. We show that in many cases these matrices will have low-rank off-diagonal structures which can be exploited to create solvers that are competitive with classical methods that yield sparse matrices.

Hyperexpansivity version of the Berger-Shaw theorem

Sameer Chavan

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A bounded linear operator T is 2-hyperexpansive if

$$I - 2T^*T + {T^*}^2T^2 \leq 0.$$

Finitely multi-cyclic 2-hyperexpansive operators admit trace-class self-commutators. Indeed, for any 2-hyperexpansive T,

$$\mathrm{trace}[T^*, T] \ \leq \ \frac{m}{\pi}\mathrm{Area}(\sigma(T)),$$

where m denotes the dimension of the null-space of T^*. As a consequence, one can conclude that every analytic 2-hyperexpansion with finite-dimensional cokernel is unitarily equivalent to a compact perturbation of a unilateral shift.

Magic in non-commutative computation

Man-Duen Choi

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Suddenly, it comes to the era of quantum computers, where non-commutative matrix analysis will play a central role in applications. Herein, I will show the hidden truth/myth in some sorts of magical computation, in connections to operator inequalities.

Convergence study on Gross-Pitaevskii equation

Yung-Sze Choi

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In this talk I will review some of my previous works with I. Koltracht. The works are on numerical solutions of some semilinear elliptic equations, in particular the Gross-Pitaevskii equation, $-\Delta u + V(\mathbf{x})u + ku^3 = \lambda u$, will be used as an example. Here k and λ are given positive constants and V is a given non-negative function. Existence of multiple solutions and convergence analysis to the discretized equations are studied. It is surprising to find that starting with an initial guess far away from the final solution, Newton's method can be shown to convergence monotonically under suitable circumstances.

Factorization of a J-selfadjoint convection-diffusion operator

Marina Chugunova

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The time evolution of a thin film of liquid on the inner surface of a cylinder rotating in a gravitational field can be described by the forward-backward heat equation:

$$h_t + Lh = 0, \quad \theta \in (-\pi, \pi), \quad t \in (0, T),$$

where

$$Lh = \varepsilon\, \partial_\theta (\sin\theta\, h_\theta) + h_\theta, \quad h(-\pi) = h(\pi), \quad \varepsilon > 0.$$

We prove that the non-selfadjoint differential operator L admits factorization. We use this factorization to construct explicitly its domain and to prove that this operator is J-selfadjoint in some Krein space. We also discuss recent progress in analysis of spectral properties of the operator L.

This talk is based on a paper with V. Strauss and on joint results with I. Karabash, S. Pyatkov.

Borg-Marchenko-type uniqueness results for CMV operators with matrix-valued Verblunsky coefficients

Stephen Clark

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Basic Weyl-Titchmarsh theory for CMV (Cantero, Moral, and Velazquez) operators with matrix-valued Verblunsky coefficients is discussed as well as local and global versions of Borg-Marchenko-type uniqueness theorems for half-lattice and full-lattice CMV operators with matrix-valued Verblunsky coefficients. Half-lattice results are formulated in terms of matrix-valued Weyl-Titchmarsh functions, while full-lattice results involve the diagonal and main off-diagonal Green's matrices. This is a joint work with F. Gesztesy and M. Zinchenko.

Factorization algorithm for some classes of Hermitian matrix functions

Ana C. Conceição (presenter) and Viktor G. Kravchenko

Department of Mathematics, University of Algarve, Faro, Portugal

We construct an algorithm (that is a development of the work Conceição and Kravchenko (2007), (2008)) that allows us to determine an effective factorization of Φ-factorable Hermitian matrix functions with elements belonging to the class L_∞ and with one of the diagonal entries preserving the sign almost everywhere on the unit circle (or on the real line) and invertible in L_∞. For those matrix functions whose entries can be represented through an inner-outer factorization (when the outer function is rational) it is shown that its explicit factorization can be obtained through the solutions of two non-homogeneous equations.

Eigenvalue problems with boundary conditions depending polynomially on the eigenparameter

Branko Curgus

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Let S be a closed densely defined symmetric operator with equal defect numbers $d < \infty$ in a Hilbert space $\big(\mathfrak{H}, \langle\,\cdot\,,\cdot\,\rangle\big)$. Let $\mathsf{b} : \mathrm{dom}(S^*) \to \mathbb{C}^{2d}$ be a boundary mapping for S. Let $\mathcal{P}(z)$ be a $d \times 2d$ matrix polynomial. We will give sufficient conditions on $\mathcal{P}(z)$ under which the eigenvalue problem

$$S^* f = \lambda f, \quad \mathcal{P}(\lambda)\mathsf{b}(f) = 0$$

is equivalent to an eigenvalue problem for a self-adjoint operator $\widetilde{A}$ in a Pontryagin space which is the direct sum of $\mathfrak{H}$ and a finite-dimensional space. The main tool in the construction of $\widetilde{A}$ is a Bezoutian matrix associated with the polynomial $\mathcal{P}(z)$.

Spectral obstructions to lifting for commuting pairs of subnormal operators

Raul Curto (presenter) and Jasang Yoon
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We study the Lifting Problem for Commuting Subnormals, especially the existence of spectral obstructions to lifting. We show how the spectral picture of hyponormal 2-variable weighted shifts with commuting subnormal components can be used to describe such an obstruction. By contrast with all known results in the theory of (single and 2-variable) weighted shifts, we show that the Taylor essential spectrum can be disconnected. We do this by obtaining a simple sufficient condition that guarantees disconnectedness, based on the norms of the horizontal slices of the shift. We also show that for every $k \geq 1$ there exists a k-hyponormal 2-variable weighted shift whose horizontal and vertical slices have 1- or 2-atomic Berger measures, and whose Taylor spectrum is disconnected. We use tools and techniques from multivariable operator theory, from our previous work on the Lifting Problem for Commuting Subnormals, and from the groupoid machinery developed by the speaker and P. Muhly to analyze the structure of C^*-algebras generated by multiplication operators on Reinhardt domains. As a by-product, we show that, for 2-variable weighted shifts, the Taylor essential spectrum is not necessarily the boundary of the Taylor spectrum.

An analogue of the Riesz-Haviland Theorem for the truncated moment problem

Raul Curto
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Let $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{|i|\leq 2n}$ denote a d-dimensional real multisequence, let K denote a closed subset of $\mathbb{R}^d$, and let $\mathcal{P}_{2n} := \{p \in \mathbb{R}[x_1, \ldots, x_d] : \deg\ p \leq 2n\}$. Corresponding to β, the *Riesz functional* $L \equiv L_\beta : \mathcal{P}_{2n} \longrightarrow \mathbb{R}$ is defined by $L(\sum a_i x^i) := \sum a_i \beta_i$. We say that L is K*-positive* if whenever $p \in \mathcal{P}_{2n}$ and $p|_K \geq 0$, then $L(p) \geq 0$. In joint work with Lawrence A. Fialkow, we prove that β admits a K-representing measure if and only if L_β admits a K-positive linear extension $\tilde{L} : \mathcal{P}_{2n+2} \longrightarrow \mathbb{R}$. This provides a generalization (from the full moment problem to the truncated moment problem) of the Riesz-Haviland Theorem. We also show that a semialgebraic set solves the truncated moment problem in terms of natural "degree-bounded" positivity conditions if and only if each polynomial strictly positive on that set admits a degree-bounded weighted sum-of-squares representation.

The Hilbert–Dirac operator on $\mathbf{R}^{2n}$ in Hermitean Clifford analysis

Hennie De Schepper

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Hermitean Clifford analysis is a recent branch of Clifford analysis, refining the standard Euclidean case. It focuses on the simultaneous null solutions, called Hermitean monogenic functions, of two complex Dirac operators which are invariant under the action of the unitary group. The specificity of the framework, introduced by means of a complex structure creating a Hermitean space, forces the underlying vector space to be even dimensional.

In engineering sciences, and in particular in signal analysis, the Hilbert transform of a real signal $u(t)$ of a one-dimensional time variable t has become a fundamental tool. The multidimensional approach to the Hilbert transform usually is a tensorial one, considering the so-called Riesz transforms in each of the variables separately. As opposed to these tensorial approaches, Clifford analysis is particularly suited for a treatment of multidimensional phenomena, encompassing all dimensions at the same time as an intrinsic feature.

In this contribution, we devote ourselves to the introduction of a Hilbert transform on $\mathbf{R}^{2n}$ in the Hermitean setting. Due to the forced even dimension of all vector spaces involved, any Hilbert convolution kernel in $\mathbf{R}^{2n}$ should originate from the non-tangential boundary limits of a corresponding Cauchy kernel in $\mathbf{R}^{2n+2}$. We show that the difficulties posed by this inevitable dimensional jump can be overcome by following a matrix approach. The resulting matrix Hermitean Hilbert transform also gives rise, through composition with the matrix Dirac operator, to a Hermitean Hilbert-Dirac convolution operator factorizing the Laplacian and being closely related to Riesz potentials.

Universality for mathematical and physical systems

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All physical systems in equilibrium obey the laws of thermodynamics. In other words, whatever the precise nature of the interaction between the atoms and molecules at the microscopic level, at the macroscopic level, physical systems exhibit universal behavior in the sense that they are all governed by the same laws and formulae of thermodynamics.

In the talk the speaker will recount some recent history of universality ideas in physics starting with Wigner's model for the scattering of neutrons off large nuclei and show how these ideas have led mathematicians to investigate universal behavior for a variety of mathematical systems. This is true not only for systems which have a physical origin, but also for systems which arise in a purely mathematical context such as the Riemann hypothesis, and a version of the card game solitaire called patience sorting.

Model reduction in symbolically semi-separable systems with application to building pre-conditioners for 3D sparse systems of equations

Patrick Dewilde (presenter) and H. Jiao

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Preconditioned iterative solvers are considered to be one of the most promising methods for solving large and sparse linear systems of equations. A central but essential problem with their use is, however, the determination of a good pre-conditioner - being an approximation to the inverse of the original matrix, which is easy to compute (with low computational complexity) and whose matrix-vector product also has low computational complexity. It has been shown in the literature that for systems that model 2D problems pre-conditioners based on semi-separable system theory can be obtained fairly easily and perform well. However, the 2D theory does not extend simply to 3D. In the presentation, we propose and evaluate a new type of preconditioners for the class of matrices that have a two level deep 'symbolically hierarchical semi-separable form' meaning that the matrices have a semi-separable-like block structure with blocks that are (sequentially) semi-separable themselves - i.e. the situation which occurs in 3D modeling problems. The new preconditioners are based on approximations of Schur complements in a sequential or hierarchical decomposition of the original block matrix. These ideas are partially inspired by the seminal work of Koltracht, Gohberg and Kailath on semi-separable systems.

The semiclassical modified nonlinear Schrödinger equation

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We study an integrable modification of the focusing nonlinear Schrödinger equation from the point of view of semiclassical asymptotics. We analyze the associated spectral problem and obtain bounds for the discrete spectrum, generalizing known results for the spectrum of the nonselfadjoint Zakharov-Shabat spectral problem. Additionally, we solve the this spectral problem in terms of special functions for a multiparameter family of initial data for all values of the semiclassical parameter. These results are viewed as part of an ongoing project analyzing the semiclassical asymptotics of modified nonlinear Schrödinger equation using the steepest descent techniques for oscillatory Riemann-Hilbert problems first developed by Deift and Zhou.

Corona conditions in a strip and invertibility for a class of Toeplitz operators

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A Toeplitz operator with symbol G such that $\det G = 1$ is invertible if there is a non-trivial solution to a Riemann-Hilbert problem $G\phi_+ = \phi_-$ with ϕ_+ and ϕ_- satisfying the corona conditions in $\mathbb{C}^+$ and $\mathbb{C}^-$, respectively. However, determining such a solution and verifying that the corona conditions are satisfied are in general difficult problems. In this talk, we present conditions on $\phi_\pm$ which are equivalent to the corona conditions but are easier to verify, if $G^{\pm 1}$ are analytic and bounded in a strip, and we identify new classes of symbols G for which a non-trivial solution to $G\phi_+ = \phi_-$ can be explicitly determined and the corona conditions can be verified by the above mentioned approach, thus obtaining invertibility criteria for the associated Toeplitz operators. This is joint work with C. Câmara.

Deviations of Riesz projections of Hill operators with singular potentials

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This talk is is based on joint results with Boris Mityagin [1-3]. The Hill operators $Ly = -y'' + v(x)y,\ x \in [0,\pi]$, with H^{-1} periodic potentials, considered with periodic, antiperiodic or Dirichlet boundary conditions (bc), have discrete spectrum, and therefore, for sufficiently large N, the Riesz projections

$$P_n = \frac{1}{2\pi i}\int_{C_n}(z - L_{bc})^{-1}dz, \quad C_n = \{z:\ |z - n^2| = n\},$$

are well defined for $n > N$. We estimate in different norms the deviations $P_n - P_n^0$ of Riesz projections, where P_n^0 are the Riesz projection of the free operator. In particular, it is proved that

$$\|P_n - P_n^0\|_{L^1\to L^\infty} \to 0 \quad \text{as } n \to \infty.$$

Our approach is based on Fourier method and techniques developed in [1] (to study Hill operators with singular potentials) and Perturbation Theory of operators. Using the same approach we give an alternative proof of the following result of A. Savchuk and A. Shkalikov [4]:

$$\sum_{n>N}\|P_n - P_n^0\|^2_{L^2\to L^2} < \infty.$$

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On dilation and extension problems on ordered groups

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Inspired by a notion given in [5], positive definite Toeplitz-Kreĭn-Cotlar triplets on ordered groups were introduced in [4]. The main result of this paper is a general dilation theorem for these triplets. In this talk this result will be presented. This general result includes and extends previous generalizations of the Kreĭn extension theorem, the Sz.-Nagy and Foias commutant lifting theorem and the Herglotz-Bochner-Weil theorem. As a corollary we present the extensions of the Nehari theorem and of the Sarason commutation theorem given in [6] for compact abelian groups whose dual have a complete linear order compatible with the group structure. The talk is based on joint work with R. Bruzual.

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Some remarks on essentially reductive Hilbert modules

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In the last several years, several authors have investigated various possible generalizations of the Berger-Shaw phenomenon, largely motivated by a conjecture of Arveson concerning the closure of homogeneous ideals in the m-shift Hilbert module over the polynomials in m variables. The best results are due to Guo and Wang. In this talk we will describe various relations between versions of a Berger-Shaw-type result for essential spherical isometries and these conjectures as well as relations between the result for homogeneous ideals and those for quasi-homogeneous ideals. Moreover, we establish all these questions for the $n = 2$ which is a modest generalization of a small part of results of Guo and Wang. This reports joint work with Jaydeb Sarkar.

On the Maxwell system

Roland Duduchava

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We will consider the Maxwell system

$$\begin{cases} \mathbf{curl}\, \boldsymbol{H} + i\omega\varepsilon \boldsymbol{E} = 0\,, \\ \mathbf{curl}\, \boldsymbol{E} - i\omega\mu \boldsymbol{H} = 0\,, \end{cases} \qquad \text{in} \quad \Omega \subset \mathbb{R}^3 \quad \text{or in} \quad \Omega^c := \mathbb{R}^3 \setminus \Omega,$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are 3-vectors, representing electric and magnetic fields, which governs interaction of time harmonic electromagnetic waves with media, occupying the domain Ω. If the media is anisotropic, the coefficients $\varepsilon = \big[\varepsilon_{jk}\big]_{3\times 3}$ (relative dielectric permittivity) and $\mu = \big[\mu_{jk}\big]_{3\times 3}$ (relative magnetic permeability) are matrices. The constant $\omega > 0$ represents the frequency. From the energy conservation law follows that these matrices are positive definite.

The feature which distinguishes the Maxwell system is that it is not elliptic and even hypoelliptic. Nevertheless, the fundamental solution exists and the potential method is applicable, although the corresponding potential operators have rather different properties than in cases of elliptic operators.

We will also discuss uniqueness of solutions of Maxwell system (Silver-Müller radiation condition at infinity) and exact solutions in case of a stratified general bianisotropic medium.

Orthogonal matrix polynomials which are eigenfunctions of differential operators

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The theory of matrix valued orthogonal polynomials was started by M. G. Krein in 1949 [3, 4] (they can be characterized as solutions of the difference equation

$$tP_n(t) = A_{n+1}P_{n+1}(t) + B_nP_n(t) + A_n^*P_{n-1}(t),$$

where A_n and B_n are, respectively, nonsingular and Hermitian matrices). But more than 50 years have been necessary to see the first examples of orthogonal matrix polynomials $(P_n)_n$ satisfying second order differential equations of the form

$$P_n''(t)F_2(t) + P_n'(t)F_1(t) + P_n(t)F_0 = \Gamma_nP_n(t).$$

Here F_2, F_1 and F_0 are matrix polynomials (which do not depend on n) of degrees less than or equal to 2, 1 and 0, respectively (see [1, 2]). These families of orthogonal matrix polynomials are among those that are likely to play in the case of matrix orthogonality the role of the classical families of Hermite, Laguerre and Jacobi in the case of scalar orthogonality.

The purpose of this talk is to show an overview of the examples found during the last five years. In particular we will discuss some of the many differences among the matrix and the scalar case, such as the (non) uniqueness of both, the second order differential operator and the sequence of orthogonal polynomials.

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Degree bounds on polynomials of symmetric matrices

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The theory of finite dimensional reproducing kernel Krein spaces is exploited to obtain formulas for the number of zeros of the determinant of an $m \times m$ matrix polynomial inside and outside the open unit disk and the open upper half-plane, assuming no zeros on the boundary. The formulas are expressed in terms of the signature of an associated matrix that coincides with the Bezoutian in the sense of Haimovici and Lerer for appropriately chosen realizations of the polynomials under consideration.

This talk is based on joint work with Dan Volok. It will be expository.

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Zero distribution of continuous analogues of matrix polynomials

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Basic properties of continuous analogues of matrix orthogonal polynomials will be reviewed and another proof of a theorem of Ellis, Gohberg and Lay that is based on reproducing kernel Pontryagin spaces of the de Branges type will be discussed. The talk will be expository.

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Invertibility theory for some classes of Toeplitz plus Hankel operators and applications

Torsten Ehrhardt

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I will discuss invertibility for Toeplitz plus Hankel operators $T(a) + H(b)$ acting on the Hardy space H^p with piecewise continuous (scalar) symbols a, b. In contrast to the Fredholm theory, invertibility theory (i.e., explicit necessary and sufficient criteria for invertibility) for such operators in general seems to be as intractable as the Wiener-Hopf factorization of 2×2 matrix-valued functions. Under the additional assumption that $a(e^{ix})a(e^{-ix}) = b(e^{ix})b(e^{-ix})$, invertibility criteria become more explicit. This class of symbols is of some importance in view of applications, which will be briefly discussed.

Quasiseparable matrices and discrete systems with boundary conditions

Yuli Eidelman (presenter) and Israel Gohberg

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A matrix is called quasiseparable of order (n_1, n_2) if its submatrices from the strictly lower triangular part are of rank n_1 at most and submatrices from the strictly upper triangular part are of rank n_2 at most. For numerical reasons one can treat quasiseparable matrices as matrices of the input output operators of linear discrete descriptor systems with boundary conditions. This reduction allows to obtain linear complexity algorithms for multiplication of a quasiseparable matrix by a vector, product of quasiseparable matrices and quasiseparable matrix inversion.

Factorization of the ccattering matrix and the location of the eigenvalues of the Manakov-Zakharov-Shabat system

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We show how the scattering matrix associated to the Manakov-Zakharov-Shabat (MZS) system can be factorized as the product of two scattering matrices associated to the Zakharov-Shabat (ZS) system of the Nonlinear Schrödinger (NLS) equation, whenever the initial conditions of the Manakov system have disjoint support. Moreover, if these initial conditions are assumed to be single-lobe, the eigenvalues of the MZS system are purely imaginary.

The left tangential operator-argument interpolation problem on the ball (commutative or noncommutative)

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In this talk, we show how the grassmannian approach to the classical Nevanlinna-Pick interpolation problem due to Ball-Helton can be adapted to arrive at the solutions (especially the parametrization of the set of all solutions) of the left tangential operator-argument interpolation problems in multivariable settings. We will focus on the interpolation problems for Schur multiplier class of Drury-Arveson space and noncommutative Schur multiplier class of Fock Space. The main tool is an indefinite-metric analogue of recent work of Ball, Bolotnikov and the speaker on Drury-Arveson space and Fock Space.

Regional information capacity of the linear, time-varying channel

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One of the fundamental theorems of information theory is Shannon's capacity of the linear time-invariant channel. We present recent work on the capacity of the linear *time-varying* channel. We view the channel as a Weyl pseudodifferential operator and show how one can construct orthonormal Weyl-Heisenberg signaling sets that are approximate channel eigenfunctions. The channel is then an operator acting on the span of a finite subset of signals. We approximate the singular values of this operator by samples of the Weyl symbol of the operator composed with its adjoint. We are thus able to derive an approximation to the information capacity of the system from the channel's Weyl symbol.

Abstract vs. concrete solutions to the truncated moment problem

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We discuss different levels of concreteness in solutions of the truncated multivariable K-moment problem, where representing measures are required to be supported in a prescribed closed subset K of $\mathbb{R}^d$. Hitherto, concrete solutions, adequate for solving numerical problems, were known only for $K = \mathbb{R},\ [0, +\infty),\ [a, b]$, or when K is a planar curve of degree at most 2. A completely general, but abstract, solution (proved in collaboration with Raúl Curto), generalizes the classical Riesz-Haviland Theorem, and expresses the existence of K-representing measures in terms of extensions of K-positive linear functionals. This solution is most useful if the positive polynomials on K admit degree-bounded weighted sum-of-squares decompositions. We apply this approach when K is an algebraic curve of the form $y = g(x)$ or $yg(x) = 1$ ($g \in \mathbb{R}[x]$). Another abstract solution expresses conditions for K-representing measures in terms of flat extensions of positive moments matrices and localizing matrices. In the case when K is the curve $y = x^3$, we show how this solution can be adapted to yield a concrete solution.

Properties of Perron-Frobenuis matrix polynomials

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In this talk we consider (spectral) properties of Perron-Frobenius matrix polynomials, i.e. polynomials $P(\cdot)$ such that

$$P(\lambda) = \lambda^m - A(\lambda) = \lambda^m - (\lambda^l A_l + ... + \lambda A_1 + A_0),$$

where the coefficients A_j are entrywiswe nonnegative square matrices. Essential for the investigations is the log-log convexity of

$$[0, \infty[\to [0, \infty[\quad \text{with} \quad \rho \mapsto \text{spectral radius of } A(\rho).$$

For $0 < m < l$ the polynomial $P(\cdot)$ is the product of a comonic Perron-Frobenius polynomial of degree $l - m$ and a monic Perron-Frobenius polynomial of degree m if and only if

$$\text{spectral radius of } A(\rho) \ < \ \rho^m \quad \text{for some positive} \quad \rho.$$

If the sum of the coefficients is a irreducible matrix then we prove an analogue of the well known result about the cyclicity of the peripheral eigenvalues of a nonnegative irreducible matrix. For $0 < m < l$ there are exactly 8 different possibilities for the existence of a monic Perron-Frobenius factor of degree m of the polynomial $P(\cdot)$ and its spectral properties.
Perron-Frobenius polynomials are equivalent to

$$\lambda - (\lambda^{l-m} \hat{A}_{l-m} + ... + \hat{A}_0),$$

therefore we can reduce some of our considerations to polynomials of the latter type.
This is a joint work with N. Hartanto (Technical University Berlin) and B. Nagy (Technical University Budapest).

Contractive inversion formulas of Gohberg-Heinig type

Arthur Frazho

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In this talk we will present Gohberg-Semencul and Gohberg-Heinig inversion type formulas for certain contractions. These inversion formulas exploit the defect subspaces for a contraction to establish the Gohberg-Heinig type inversion formula. An application to Toeplitz plus Hankel operators, and $S(m)$ models will be given. Finally, we will also use these inversion formulas with Lyapunov techniques, to develop some inversion formulas for operators naturally arising in system theory and signal processing. This talk is based on joint work with M.A. Kaashoek.

A numerical method for solving the relaxed Sarason problem

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The relaxed commutant lifting theorem is used to numerically compute a solution to a relaxed version of the Sarason interpolation problem. This numerical method is based on state space, the Fast Fourier transform and the singular value decomposition. Our algorithm works for both rational and nonrational functions. As a special case, we also obtain a method to compute a solution to the classical H^∞ Sarason problem. This talk is based on joint work with D. Dzu.

Tensor products of functional models, the Sylvester equation and Bezoutians

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We study tensor products of vectorial polynomial models over the underlying field $\mathbb{F}$ as well as over the ring of polynomials $\mathbb{F}[z]$ and derive concrete, functional, representations for these products. This leads to a polynomial version of the Sylvester equation, to a new approach to the characterization of intertwining maps for polynomial models and, finally, to a basis free approach to generalized Bezoutians. Some other applications will be indicated.

Matrix trace inequalities on a generalized Wigner-Yanase skew information

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Matrix trace inequalities on a generalized Wigner-Yanase skew information Abstract:We introduce a generalized Wigner-Yanase skew information and then derive the trace inequality related to the uncertainty relation. This inequality is a non-trivial generalization of the uncertainty relation derived by S.Luo for the quantum uncertainty quantity excluding the classical mixure. In addition, several trace inequalities on our generalized Wigner-Yanase skew information are argued. This is joint work with Ken Kuriyama and Kenjiro Yanagi.

Further extension of order preserving operator inequality and its application

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A capital letter means a bounded linear operator on a Hilbert space H. The celebrated Löwner-Heinz inequality asserts that *if* $A \geq B \geq 0$*, then* $A^\alpha \geq B^\alpha$ *holds for any* $\alpha \in [0,1]$*, and* $A^\alpha \geq B^\alpha$ *does not always hold for* $\alpha > 1$. The following result has been obtained from this point of view.

Theorem A. *If* $A \geq B \geq 0$*, then for each* $r \geq 0$*,*

$$\text{(i)} \quad (B^{\frac{r}{2}} A^p B^{\frac{r}{2}})^{\frac{1}{q}} \geq (B^{\frac{r}{2}} B^p B^{\frac{r}{2}})^{\frac{1}{q}} \quad \textit{and} \quad \text{(ii)} \quad (A^{\frac{r}{2}} A^p A^{\frac{r}{2}})^{\frac{1}{q}} \geq (A^{\frac{r}{2}} B^p A^{\frac{r}{2}})^{\frac{1}{q}}$$

hold for $p \geq 0$ *and* $q \geq 1$ *with* $(1+r)q \geq p+r$*.*

The following Theorem B is an extension of Theorem A.

Theorem B. *If* $A \geq B \geq 0$ *with* $A > 0$*, then for* $t \in [0,1]$ *and* $p \geq 1$*,*

$$F(r,s) = A^{\frac{-r}{2}} \{A^{\frac{r}{2}} (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^s A^{\frac{r}{2}}\}^{\frac{1+r-t}{(p-t)s+r}} A^{\frac{-r}{2}}$$

is a decreasing function for $r \geq t$ *and* $s \geq 1$*, and*

$$A^{1+r-t} \geq \{A^{\frac{r}{2}} (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^s A^{\frac{r}{2}}\}^{\frac{1+r-t}{(p-t)s+r}}$$

holds for $t \in [0,1]$*,* $p \geq 1$*,* $r \geq t$ *and* $s \geq 1$*.*

In this talk, we show Theorem C which is further extension of Theorem B and its application.

Theorem C. *Let $A \geq B \geq 0$ with $A > 0$, $t \in [0,1]$ and $p_1, p_2, ..., p_{2n} \geq 1$ for natural number n. Then*
$G_{A,B}[r, p_{2n}] =$

$$A^{\frac{-r}{2}} \left\{ A^{\frac{r}{2}} \Big[\underbrace{A^{\frac{-t}{2}} \{A^{\frac{t}{2}} ......[A^{\frac{-t}{2}} \{A^{\frac{t}{2}} (A^{\frac{-t}{2}}}_{\leftarrow A^{\frac{-t}{2}} \text{ } n \text{ times and } A^{\frac{t}{2}} \text{ } n-1 \text{ times by turns}} B^{p_1} \underbrace{A^{\frac{-t}{2}})^{p_2} A^{\frac{t}{2}}\}^{p_3} A^{\frac{-t}{2}}]^{p_4} A^{\frac{t}{2}} ....A^{\frac{-t}{2}}}_{\rightarrow A^{\frac{-t}{2}} \text{ } n \text{ times and } A^{\frac{t}{2}} \text{ } n-1 \text{ times by turns}} \Big]^{p_{2n}} A^{\frac{r}{2}} \right\}^{\frac{1+r-t}{q[2n]+r-t}} A^{\frac{-r}{2}}$$

is a decreasing function of $p_{2n} \geq 1$ and $r \geq t$, and the following inequality holds

$$G_{A,A}[r, p_{2n}] \geq G_{A,B}[r, p_{2n}],$$

that is,

$$A^{1+r-t} \geq \left\{ A^{\frac{r}{2}} \left[\underbrace{A^{\frac{-t}{2}} \{A^{\frac{t}{2}} ....[A^{\frac{-t}{2}} \{A^{\frac{t}{2}} (A^{\frac{-t}{2}}}_{\leftarrow A^{\frac{-t}{2}} \text{ } n \text{ times and } A^{\frac{t}{2}} \text{ } n-1 \text{ times by turns}} B^{p_1} \underbrace{A^{\frac{-t}{2}})^{p_2} A^{\frac{t}{2}}\}^{p_3} A^{\frac{-t}{2}}]^{p_4} A^{\frac{t}{2}} ....A^{\frac{-t}{2}}}_{\rightarrow A^{\frac{-t}{2}} \text{ } n \text{ times and } A^{\frac{t}{2}} \text{ } n-1 \text{ times by turns}} \right]^{p_{2n}} A^{\frac{r}{2}} \right\}^{\frac{1+r-t}{q[2n]+r-t}}$$

where $q[2n] = q[2n; p_1, p_2, ..., p_{2n}] = \underbrace{\Big\{.....[\{[(p_1 - t)p_2 + t]p_3 - t\}p_4 + t]p_5 - ...... - t\Big\}p_{2n} + t}_{\text{-t and t alternately } n \text{ times appear}} =$

$$\prod_{i=1}^{2n} p_i + \underbrace{(\prod_{i=3}^{2n} p_i + \prod_{i=5}^{2n} p_i + ... + \prod_{i=7}^{2n} p_i + ... + p_{2n-1}p_{2n} + 1)}_{\text{n terms}} t - \underbrace{(\prod_{i=2}^{2n} p_i + \prod_{i=4}^{2n} p_i + \prod_{i=6}^{2n} p_i + ... + p_{2(n-1)}p_{2n-1}p_{2n} + p_{2n})}_{\text{n terms}} t.$$

We discuss applications of Theorem C, especially transformation of Theorem C into log majorization.

Unitary equivalence to a complex symmetric matrix

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It is well-known that any $n \times n$ complex matrix is *similar* to a complex symmetric matrix (CSM) (i.e., $A = A^t$). Indeed, as opposed to a selfadjoint matrix (i.e., $A = A^*$), a CSM can have any possible Jordan form. This makes the problem of determining exactly which matrices / operators are *unitarily equivalent* to a complex symmetric matrix (UECSM) somewhat difficult. In particular, we cannot employ any of the standard similarity invariants from linear algebra. For instance the following matrices are obviously all similar, but exactly one is UECSM:

$$\begin{pmatrix} 0 & 7 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 6 \end{pmatrix} \begin{pmatrix} 0 & 7 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 6 \end{pmatrix} \begin{pmatrix} 0 & 7 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 6 \end{pmatrix} \begin{pmatrix} 0 & 7 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 6 \end{pmatrix}$$

(Can you guess which one?). We discuss several recent solutions to this problem. To this end, we also discuss the basic structure of *complex symmetric operators* on Hilbert space, highlighting some surprising members of this family, and a few applications.

Positive definite locally Hilbert space operator-valued kernels

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The role played by the positive definiteness of Hilbert space operator valued kernels in the development of various branches of mathematics is widely known. Let us only recall the non-selfadjoint spectral theory initiated by the famous dilation theorem of B. Sz.-Nagy, the harmonic analysis of multivariate stochastic processes started by the works of N. Wiener and P. Masani, as well as the theory of Hilbert modules over C^* - algebras (R. Kaplanski, W. Pashke). Aiming to cover in an analogue unitary manner some of the recent developments such as dilation of Banach space operators, the study of infinite variate and generalized Banach space valued stochastic processes and dilations of Hilbert C^* - module operators, we have to revert to the development of a local theory of positive definiteness. As such we have recently studied the positive definiteness for locally C^* - valued kernels and alikes and introduced the related reproducing kernel Hilbert modules.

In the present work we restrain ourselves to the standard locally C^* - algebra $\mathcal{L}(\boldsymbol{\mathcal{H}})$ associated to a locally Hilbert space $\boldsymbol{\mathcal{H}}$, which is an inductive limit of some ascending family of Hilbert spaces.

For an $\mathcal{L}(\boldsymbol{\mathcal{H}})$ - valued positive definite kernel Γ, besides of the above mentioned reproducing kernel Hilbert $\mathcal{L}(\boldsymbol{\mathcal{H}})$ - module $\mathfrak{H}_\Gamma$ we also define a reproducing kernel locally Hilbert space $\boldsymbol{\mathcal{K}}_\Gamma$ (of $\mathcal{L}(\boldsymbol{\mathcal{H}})$ - valued functions). After proving the basic property of $\boldsymbol{\mathcal{K}}_\Gamma$ to be an inductive limit of a family of reproducing kernel Hilbert spaces, the following types of results are obtained

(1) The construction of an operatorial model for the reproducing kernel Hilbert $\mathcal{L}(\boldsymbol{\mathcal{H}})$ - module, in terms of $\boldsymbol{\mathcal{H}}$ and $\boldsymbol{\mathcal{K}}_\Gamma$.

(2) A Sz.-Nagy type dilation theorem for a positive definite $\mathcal{L}(\boldsymbol{\mathcal{H}})$ - valued function on a $*$ - semigroup, which satisfies a "locally" boundedness condition.

(3) From the locally non-selfadjoint spectral theory we mention dilations of locally adjointable contractions or of positive $\mathcal{L}(\boldsymbol{\mathcal{H}})$ - valued measures, as well as extensions for locally isometries or for locally subnormal operators.

The shift of a random distribution

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The efficiency of operator theoretical methods in many branches of applied mathematics is well known. Let's only recall the one time parameter continuous stationary second order stochastic processes (i.e. $L^2(p)$ - valued, with $(\Omega, \mathcal{A}, p)$ a probability space) to which a unitary group (on the real line) as shift of the process can be attached. In this way the spectral theory for unitary groups on $L^2(p)$ enables us to give an elegant treatment of the harmonic analysis of such stochastic processes. The spectral theory on unitary hilbertian representations of locally compact groups allows a similar treatment of the strong second order random fields of finitely many continuous time parameters.

Aiming to cover more concrete situations from very different domains, sometimes it is needed to regard a random field not only as a continuous function but as an $L^2(p)$ - valued L. Schwartz distribution (e.g. the Brownian motion).

We call such an object *strong second order random field distribution*, dropping the word "field" in the case of one time continuous parameter ($n = 1$).

In this talk, based on joint research with D. Gaşpar, N. Grindeanu and L. Popa, we define and study a stationary random field distribution S with n continuous time parameters by associating to each such S an appropriate shift V^S, which is an operator (on $L^2(p)$) valued distribution or, more precisely, a unitary group distribution (on $\mathbb{R}^n$) in the sense of J.L. Lions and C. Foiaş.

After giving some basic properties of S and V^S, we give an integral representation of V^S in terms of the Laplace-Fourier transform on $\mathbb{R}^n$ and of a spectral measure E_S on $\mathbb{C}^n$ and then we show that E_S is in fact supported on $\mathbb{R}^n$ and V^S is a regular (continuous) Hilbert space operator valued distribution. This means finally that V^S can be viewed as a unitary group on $\mathbb{R}^n$, which leads to expected results in the harmonic analysis of strong second order stationary random field distribution.

In the multivariate case, as it is well outlined in the book of Yu. Kakihara, the space of $\mathcal{E}$ - valued ($\mathcal{E}$ a Hilbert space) random variables is organized as a normal Hilbert $\mathcal{B}(\mathcal{E})$ - module, $\mathcal{C}_2(\mathcal{E}, L^2(p))$. This means that a similar treatment of the second order stochastic processes in this frame, presume a development and an application of a harmonic analysis with $\mathcal{C}_2(\mathcal{E}, L^2(p))$ - valued or operator (on such Hilbert modules) - valued distributions. In such a way it is shown in the last part that the results regarding the associated shift can be essentially extended to the multivariate second order stochastic random fields distributions.

Finite Blaschke products of numerical contractions

Hwa-Long Gau

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This is joint work with Pei Yuan Wu (National Chiao Tung University, Taiwan).

Let f be a disk algebra function with $\|f\|_\infty = 1$ and T be a bounded linear operator on a complex Hilbert space H. We prove that if $w(T) \leq 1$ and $\|f(T)x\| = 2$ for some unit vector $x \in H$, then f is a finite Blaschke product with $f(0) = 0$, and T is unitarily equivalent to an operator of the form $XS(\phi)X^{-1} \oplus T'$, where $\phi(z) = zf(z)$, $S(\phi)$ is the compression of the shift to the space $H(\phi) = H^2 \ominus \phi H^2$ and

$$X = \begin{bmatrix} \sqrt{2} & & \\ & I & \\ & & \frac{1}{\sqrt{2}} \end{bmatrix} \quad \text{on} \quad H(\phi) = \ker S(\phi) \oplus (H(f) \ominus \ker S(\phi)) \oplus \ker S(\phi)^*.$$

Thus we give an affirmative answer to a question asked by Drury, and also improve a result of Crabb in 1971.

Bivariate Orthogonal polynomials and the Christoffel-Darboux formula

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The theory of bivariate orthogonal polynomials on the bicircle has been shown to be useful for obtaining results in two complex variables. We will review this theory and present some results in this area.

An efficient solution of the the inverse boundary impedance problem in the half-space

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The inverse scattering problem of determining the boundary impedance from the value of scattering amplitude is considered. We introduce a modified potential through which the boundary impedance is explicitly expressed by means of a nonlinear relation. Then a linear system of equation is obtained for calculation of the modified potential. We also provide an efficient formula for the boundary impedance in the case of high frequency incident waves. Two and three-dimensional examples are given to demonstrate advantage of the new approach.

Resultant. The past and the present

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The classical Resultant matrix of two scalar polynomials of one variable was introduced by James Joseph Sylvester as a tool for reducing a system of two polynomial equations with two unknowns to the solution of two polynomial equations (of higher degree) each with one unknown. In the definition used here the resultant of two scalar polynomials of one variable is a square matrix whose kernel gives a complete description of the common zeros of the two initial polynomials. The talk is about recent generalizations of this result for matrix polynomials. The first step was done by I. Gohberg and G. Heinig in a 1976 paper. It was found that the Sylvester resultant formula for the general case must be modified, namely: the resultant matrix has to be replaced by a nonsquare matrix of similar structure as the resultant. This result was used for the solution of a certain inverse problem for block Toeplitz matrices. Recently I. Gohberg, M.A. Kaashoek, L. Lerer found the necessary and sufficient condition in order that the original Sylvester result can be straightforwardly generalized for the matrix polynomial case. The condition is stated in terms of a quasi commutativity property. For the Sylvester scalar case this condition is automatically satisfied. Applications to inverse problems for orthogonal polynomials will be formulated if time permits. The continual analogs of the presented results will be discussed in the plenary talk of M.A. Kaashoek.

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Fast algorithms for integral equations and Krein-Sobolev equation

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This talk will start with reminiscences about my friend and former student Israel Koltracht that recently passed away and about some of our joint work. In the talk is described a unified approach for obtaining and treatment of fast numerical algorithms for different classes of Fredholm integral equations of second kind. This approach allows to treat Hankel and semiseparable kernels, the class of displacement and close to displacement kernels and also displacement plus Hankel kernels. The Krein-Sobolev nonlinear functional-differential equation for the resolvent provides the basis for the unified approach and treatment. Convergence and stability of related numerical schemes are also presented. The talk is based on the following joint paper:

I. Gohberg and I. Koltracht, *Numerical solutions of Integral Equations, Fast algorithms and Krein-Sobolev Equations*, Numerisch Mathematik **47**, 237-288 , Springer Verlag, (1985)

LU and UL factorization of integral operators with semi-separable kernel and symmetries

Gilbert Groenewald

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In this talk we discuss LU and UL factorizations of integral operators with semi-separable kernels. The results are applied to a class of such integral operators that are positive definite. This talk is based on joint work with M.A. Petersen and A.C.M. Ran.

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Uniform boundedness of Toeplitz matrices with variable coefficientas

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Uniform boundedness of sequences of variable-coefficient Toeplitz matrices is a surprisingly delicate problem. We show that if the generating function of the sequence belongs to a smoothness scale of the Holder type and if α is the smoothness parameter, then the sequence may be unbounded for $\alpha < .05$ while it is always bounded for $\alpha > .05$

Toeplitz operators with frequency modulated almost periodic and semi-almost periodic symbols

Sergei Grudsky (presenter), A. Böttcher and I.M. Spitkovsky
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In 1968, I. Gohberg and I. Feldman worked out a theory of Toeplitz operators with almost periodic symbols (in terms of Wiener-Hopf integro-difference operators). This talk presents a generalization of their work in the direction of so-called frequency modulated symbols. These symbols are superpositions of (semi-)almost periodic functions and a wide class of homeomorphisms of the real line onto itself. On the basis of the theory of u-factorization, one considers the scalar case, the matrix case and the case of the algebras generated by Toeplitz operators with frequency modulated symbols.

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The Darboux process in the matrix valued case

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The Darboux method (introduced by Moutard) has been used extensively by many authors to produce more elaborate examples of bispectral situations out of simple ones starting with the work of H. Duistermaat and myself in the continuous-continuos case.

Slightly more recent contributions include my joint work with L. Haine, P. Iliev, E. Horozov, and M. Yakimov as well as work by J. Liberati, G. Wilson, B. Bakalov, S. Veselov, Y. Berest and a few more. This problem has unexpected connections with several parts of mathematics.

I will give a brief ab-initio introduction and then talk about some recent progress in the matrix case.

Direct and inverse problems for networks

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Consider a network with known connectivity consisting of some input nodes, some output nodes and several internal (and innaccesible nodes). Messages are sent from the incoming nodes and (maybe) received at the output ones. The underlying dynamics is modeled by a Markov chain with unknown transition probability matrix. The data is the so called traffic matrix, or input-output relation and the unkown is the transition probability matrix.

This can be seen as a nonlinear version of the problem of Network Tomography, and is motivated by previous work on Diffuse or Optical Tomography, where an infrared source is used to (try to) obtain images of attenuation and scattering characteristics of human tissue.

I will show that some simple but non trivial examples can be handled, and illustrate in some cases the use of methods from the orthogonal polynomial toolkit.

Remarks to dual Toeplitz operators

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Let $\mathbb{D}$ be the unit disk in the complex plane $\mathbb{C}$ and let $dA(z)$ be the Lebesgue measure on $\mathbb{D}$. The Lebesgue space of square summable complex-valued functions is denoted by $L^2(\mathbb{D}, dA)$. The Bergman space $L^2_a(\mathbb{D})$ is the Hilbert subspace of $L^2(\mathbb{D}, dA)$ consisting of analytic functions; its orthogonal complement is denoted by $\left(L^2_a(\mathbb{D})\right)^\perp$. The harmonic Bergman space is characterized by $L^2_h(\mathbb{D}) = L^2_a \oplus \overline{zL^2_a}$; it consists of all square integrable harmonic functions. Both $L^2_a(\mathbb{D})$ and $L^2_h(\mathbb{D})$ are reproducing kernel Hilbert spaces. For a symbol $\varphi \in L^1(\mathbb{D}, dA)$, the corresponding Toeplitz operator T_φ is defined to be the multiplication by φ followed by the projection $\mathcal{P}$, i.e. $T_\varphi : \ f \in L^2_a(\mathbb{D}) \longrightarrow T_\varphi(f) = \mathcal{P}(\varphi f) \in L^2_a(\mathbb{D})$. If the orthogonal projection on $\left(L^2_a(\mathbb{D})\right)^\perp$ is denoted by $\mathcal{Q} = I - \mathcal{P}$, then the dual Toeplitz operator can be defined by $\mathcal{S}_\varphi : \ f \in \left(L^2_a(\mathbb{D})\right)^\perp \longrightarrow \mathcal{S}_\varphi(f) = \mathcal{Q}(\varphi f) \in \left(L^2_a(\mathbb{D})\right)^\perp$. The Toeplitz and dual Toeplitz operators on the harmonic Bergman space $L^2_h(\mathbb{D})$ and its orthogonal complement $\left({}^2_h(\mathbb{D})\right)^\perp$, respectively, can be defined similarly, with $\mathcal{P}$ being the orthogonal projection from $L^2(\mathbb{D}, dA)$ onto $\left(L^2_h(\mathbb{D})\right)^\perp$ instead.

Dual Toeplitz operators in the "analytic" Bergman space case have been extensively studied by Stroethoff and Zheng in [10]. Toeplitz operators on the harmonic Bergman space have been considered for instance in [5, 6, 9, 12]. The numerical range of a Toeplitz operator has been investigated in [8, 11]. The famous fifth problem of Halmos has been posed in [7], and has been discussed in [1, 2, 4, 3]. Our aim here is two folds:

From one hand, we point out some remarks regarding the spectral properties of dual Toeplitz operators. Moreover, we investigate the numerical range of a dual Toeplitz operator in several typical cases. Besides, for dual Toeplitz operators, we shed some light on the analog of Halmos' classification problem of subnormal Toeplitz operators.

On the other hand, we introduce dual Toeplitz operators on the orthogonal complement of the harmonic Bergman space $\left(L^2_h(\mathbb{D})\right)^\perp$ and, therefore, initiate a systematic study of this important class of operators from various points of view.

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On the ideals of Orlicz type operators

Manjul Gupta (presenter) and L. R. Acharya
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We establish results on the mappings of type ℓ_M defined by approximation numbers, for a given Orlicz function M. Associating the spaces of type ℓ_M with Köthe sequence spaces we further study the operators and spaces of type h_M.

Norm inequalities for composition operators on Hardy and weighted Bergman spaces

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It is well known that any analytic self-map of the open unit disk induces a bounded composition operator on the Hardy space and the standard weighted Bergman spaces. For a particular self-map, one might wonder whether there is any meaningful relationship between the norms of the corresponding operators acting on each of these different spaces. After discussing the context of the problem, we will prove an inequality that (at least to a certain degree) answers this question. We will also consider some related questions that are still open, and discuss possible connections between this problem and the issue of cosubnormality of composition operators. (This talk is based on joint research with Linda J. Patton.)

On the complexity index and approximations of spectra and pseudospectra

Anders Hansen
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In this talk I will discuss how to recover spectra and pseudospectra of Hilbert space operators using limits of sets that can be constructed using only finitely many arithmetic operations and radicals of the matrix elements of the operator. The constructions are valid for a large set of closed operators including all the bounded ones. The tool for determining the complexity of such constructions is the complexity index. I will discuss the motivation for defining such an index and provide some explicit bounds. The theory will be accompanied by numerical examples and algorithms to show how this can be used in actual computations.

Spectra of self-adjoint extensions of a symmetric operator and its application in sampling theory

Yufang Hao (presenter) and Achim Kempf

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

In this talk, we shall examine the spectra of self-adjoint extensions of unbounded simple symmetric operators T with deficiency indices $(1,1)$. As an application, we present results on a new generalized sampling theorem.

Any $(1,1)$-symmetric operator T has a $U(1)$-family of self-adjoint extensions, say $T(\alpha)$, for $0 \leq \alpha < 2\pi$. Each self-adjoint operator $T(\alpha)$ has a set of discrete eigenvalues $\{t_n(\alpha)\}_{n=-\infty}^{\infty}$, and together they cover the real line exactly once. Given the spectrum of one self-adjoint extension and the corresponding derivatives defined as $t_n'(\alpha) = \frac{d}{d\alpha}t_n(\alpha)$, we obtain an explicit formula for computing the eigenvalues of all other self-adjoint extensions of T. This provides a computational realization of the abstract Cayley transform.

Such symmetric operators T arise in sampling theory: the set of sampling points turns out to be the point spectrum of a self-adjoint extension of T, and the reconstruction kernel corresponds to the unitary transformation, which maps between the eigenbases of the self-adjoint extensions. In classical sampling theory, which provides the link between continuous and discrete representations of information, the self-adjoint extensions are restricted case of equidistant eigenvalues. By considering generic simple $(1,1)$-symmetric operator T, we can generalize the classical sampling theory to allow non-equidistant sampling points. An explicit expression for the generalized reconstruction kernel is obtained.

Two-Isometries on Pontryagin Spaces

Chris Hellings

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We give a model for a class of cyclic analytic 2-isometries acting on Pontryagin spaces, generalizing a Hilbert space version given by S. Richter. Furthermore, we construct an example of a 2-isometry acting on a Pontryagin space of functions on the unit disk that exhibits behavior not possible in the Hilbert space setting.

Signed matrix results and applications to chemical reaction networks

Bill Helton (presenter) and Igor Klep

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A signed matrix is actually a class of matrices consisting of matrices whose ij-th entry has the same sign. There is a classical theory associating a graph to such a class and analyzing the sign of determinants of matrices in the class. We extend this to give more refined determinant results. It has been recently observed that substantial numbers of chemical reaction networks (these play a prominent role in systems biology) have dynamics $dx/dt = f(x)$ with the Jacobian of f having a sign pattern or something similar. Our results will be applied to this.

Structured matrices, continued fractions, and the generalized Routh-Hurwitz problem: Part I.

Olga Holtz

Department of Mathematics, University of California, Berkeley, CA 94720

This is joint work with Mikhail Tyaglov.

We will discuss various connections among the following topics:

- Hankel, Vandermonde, Toeplitz, Hurwitz, and Jacobi matrices;
- Decomposition of rational functions into continued fractions of Stieltjes, Jacobi and other types;
- The Routh-Hurwitz problem and its generalizations.

Part II, on applications of the general theory to root localization of polynomials, will be presented by Mikhail Tyaglov.

Redheffer representations in commutant lifting theory

Sanne ter Horst

Department of Mathematics, Virginia Tech, Blacksburg, VA 24060

Redheffer transformations play an important role in the parameterization of the solutions to interpolation and commutant lifting problems. In this talk we consider Redheffer transformations of the type appearing in relaxed and classical commutant lifting theory in a general context. We study properties of these transformations and their coefficients, and discuss, in particular, properties that determine whether a Redheffer transformation describes the solutions of a (relaxed) commutant lifting problem.

Extension of maps on operator spaces and completely preserver problems

Jinchuan Hou

Department of Mathematics, Taiyuan University of Technology; Department of Mathematics, Shanxi Normal University.

We talk about the question that under what conditions a (linear or general) map acting on a subspace (or a subset) of an operator algebra has an extension to a Jordan homomorphisms. We show that the completely rank nonincreasing linear maps acting on nest algebras or on subspaces of finite von Neumann algebras, completely invertibility preserving maps and completely spectrum preserving maps, completely idempotent preserving maps and completely square-zero preserving maps have the desired property.

Two variable measures and matrix orthogonal polynomials

Plamen Iliev

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We show that bivariate orthogonal polynomials can be linked to the theory of matrix valued orthogonal polynomials if we order the monomials with respect to lexicographical and reverse lexicographical orderings. We will consider several examples, where this connection is used to recover the two dimensional measure from its moments. The talk is based on joint work with J. Geronimo.

Extensions of the results on powers of p-hyponormal operators to class $\mathbf{wF}(p, r, q)$ operators

Masatoshi Ito

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In what follows, we shall consider bounded linear operators on a complex Hilbert space $\mathcal{H}$, and also an operator T is said to be positive (denoted by $T \geq 0$) if $(Tx, x) \geq 0$ for all $x \in \mathcal{H}$.

Recently Gao-Yang showed the following result. If T is a p-hyponormal operator (i.e., $(T^*T)^p \geq (TT^*)^p$ holds) for $0 < p \leq 1$, then

$$(T^{n+1^*}T^{n+1})^{\frac{n+p}{n+1}} \geq (T^{n^*}T^n)^{\frac{n+p}{n}} \quad \text{and} \quad (T^nT^{n^*})^{\frac{n+p}{n}} \geq (T^{n+1}T^{n+1^*})^{\frac{n+p}{n+1}}$$

hold for all positive integer n. Moreover, similar results for invertible log-hyponormal operators ($\log T^*T \geq \log TT^*$) have already shown by Yamazaki, and also it was known that Yamazaki's result holds for even class A operators ($|T^2| \geq |T|^2$ where $|T| = (T^*T)^{\frac{1}{2}}$).

In this talk, as a parallel result to that of class A operators, we shall get that above inequalities hold for weaker conditions than p-hyponomality, that is, class F(p,r,q) $\left((|T^*|^r|T|^{2p}|T^*|^r)^{\frac{1}{q}} \geq |T^*|^{\frac{2(p+r)}{q}}\right)$ defined by Fujii-Nakamoto or class wF(p,r,q) $\left((|T^*|^r|T|^{2p}|T^*|^r)^{\frac{1}{q}} \geq |T^*|^{\frac{2(p+r)}{q}} \text{ and } |T|^{2(p+r)(1-\frac{1}{q})} \geq (|T|^p|T^*|^{2r}|T|^p)^{1-\frac{1}{q}}\right)$ defined by Yang-Yuan under appropriate conditions of p, r and q.

Spectrum of block operator matrices associated to second order systems

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Cauchy problems for a second order linear differential operator equation

$$\ddot{z}(t) + A_0 z(t) + D\dot{z}(t) = 0$$

in a Hilbert space H are studied. Equations of this kind arise for example in elasticity, hydrodynamics and are used as a model for transverse motions of thin beams in the presence of damping. Here the stiffness operator A_0 is an unbounded uniformly positive operator on a Hilbert space H and D, the damping operator, is an unbounded operator, such that $A_0^{-1/2}DA_0^{-1/2}$ is a bounded accretive operator on H. This second order equation is equivalent to the standard first-order equation

$$x''(t) = Ax(t),$$

where $A : D(A) \to D(A_0^{1/2}) \times H \to D(A_0^{1/2}) \times H$, is given by

$$A = \begin{pmatrix} 0 & I \\ -A_0 & -D \end{pmatrix}.$$

This block operator matrix has been studied in the literature for more than 20 years. It is well-known that A generates a C_0-semigroup of contraction. We are interested in a more detailed study of the location of the spectrum of A. In general the (essential) spectrum of A can be quite arbitrary in the closed left half plane. Under various conditions on the damping operator D we describe the location of the spectrum and the essential spectrum of A. Further, conditions are developed guaranteeing that A generates an analytic semigroup and that there exists Riesz basis of $D(A_0^{1/2}) \times H$ consisting of generalized eigenvectors of A.

Krein Signatures for Eigenvalue Problems Associated to Integrable Systems

Mathew Johnson

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There is a result of Klaus and Shaw which shows that the Zakharov-Shabat eigenvalue problem has discrete spectrum which lies on the imaginary axis if the potential has a single critical point and decays monotonically away from this point (the potential is monomodal). We put this calculation in the context of the Krein signature (a tool for studying the stability of symplectic matrices) and prove an analogous theorem for the eigenvalue problem which solves the Sine-Gordon equation (in laboratory coordinates). This is joint work with Jared Bronski.

On the numerical solution of the notched half plane problem

Peter Junghanns

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The stability of collocation methods with respect to Chebyshev nodes for integral equations of the form

$$a(x)u(x) + \frac{b(x)}{\pi}\int_{-1} 1 \frac{u(y)\,dy}{y-x} + \int_{-1}^{1} h\left(\frac{1+x}{1+y}\right)\frac{u(y)\,dy}{1+y} = f(x)\,, \quad -1 < x < 1\,,$$

is investigated in [1]. We discuss the applicability of the results given in [1] to a hypersingular integral equation which arises from the notched half plane problem of two-dimensional elasticity theory (cf. [2, 3]).

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Operator-valued Herglotz kernels and functions of positive real part on the ball

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We describe several classes of holomorphic functions of positive real part on the unit ball; each is characterized by an operator-valued Herglotz formula. Motivated by results of J.E. McCarthy and M. Putinar, we define a family of weighted Cauchy-Fantappiè pairings on the ball and establish duality relations between certain pairs of classes. In particular we identify the dual of the positive Schur class. We also establish the existence of self-dual classes with respect to this pairing, and obtain a von Neumann-type inequality for commutative polynomials evaluated on non-commuting row contractions.

Function-theoretic aspects of Schur class mappings of the ball

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Let $\mathbb{B}^d$ denote the open unit ball of $\mathbb{C}^d$. A holomorphic mapping $\varphi : \mathbb{B}^d \to \mathbb{B}^d$ belongs to the *Schur class* if the Hermitian kernel

$$\frac{1 - \langle \varphi(z), \varphi(w) \rangle}{1 - \langle z, w \rangle}$$

is positive semidefinite. It is well known that when $d = 1$, every holomorphic self-map of the disk is Schur, but this is not so in higher dimensions. However, some theorems about self-maps of the disk which fail for general self-maps of the ball do have analogues if one restricts to the Schur class. This is particularly true of theorems which possess some operator-theoretic character, such as the Nevanlinna-Pick interpolation theorem. In this talk we consider two theorems (Littlewood's subordination theorem and the Julia-Caratheodory theorem) which we will extend to the Schur class on the ball. We will describe some applications of these results to the study of composition operators; in particular we show that every Schur class map gives a bounded composition operator on the standard function spaces, and obtain a formula for the spectral radius. We also obtain some norm estimates for weighted composition operators which are new even in one dimension.

Resultant and Bezoutian for entire matrix functions: theory and applications

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During the last five years the theory of continuous analogues of Resultant and Bezoutian for entire matrix functions, which started with the pioneering work of Gohberg and Heinig in the 70ties, has been developed further in different directions and new applications haven been added. A main new element is the description of the null space of the resultant operator, which is assumed to act between one and the same L_1-space, in terms of the common spectral data of the entire matrix functions involved. It turns out that for matrix functions such a description is possible if and only if a certain additional condition of quasi-commutativity is fulfilled. This quasi-commutativity condition allows us to use the Haimovici-Lerer analogue of the Bezoutian as a tool in the proofs. The new applications concern entire matrix function equations, inverse problems for convolution integral operators on a finite interval, inverse problems for Krein orthogonal functions and other related problems. The talk is based on joint work with Israel Gohberg and Leonid Lerer.

Connections between Clifford Analysis and co-orbit space theory

Uwe Kaehler

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Co-orbit space theory or Feichtinger/Gröchenig theory is becoming more and more a major tool in signal and image processing, mainly due to its applications to non-linear and sparse approximations. Furthermore, in recent years one can observe a growing interest in applications of Clifford analysis to signal processing, in particular to analytic signals. This interest is mainly driven by the geometric preserving properties of Clifford analysis, making it particularly suitable for applications to image processing, texture analysis and texture goniometry, and to the mathematical foundations of the Huang-Hilbert transform. In this talk we will consider co-orbit space theory from the Clifford analytic viewpoint and study the interconnections of both theories as well as their practical implications, e.g. while classic Clifford analysis provides the possibility of construction of frames using interpolating and sampling sequences, co-orbit theory allows the study of non-linear and sparse approximations in Clifford analysis.

Reproducing kernels for harmonic Besov spaces on the unit ball

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We are interested in Besov spaces b^p_q for which $q \in \mathbb{R}$ and $1 \leq p < \infty$, whose members are harmonic functions on the unit ball of $\mathbb{R}^n$ in such a way that their sufficiently high-order derivatives are in Bergman spaces of harmonic functions b^p_q, where $q > -1$. We compute the reproducing kernels of the Besov spaces b^2_q with $q \leq -1$. The kernels use natural radial fractional derivatives that are suitable for series of zonal harmonics. Using kernels, we define generalized Bergman projections and characterize those that are bounded from Lebesgue classes onto Besov spaces b^p_q. We obtain various applications of the projections. This is joint work with A. Ersin Üreyen and Seçil Gergün.

Forward-backward parabolic equations, the case of singular critical point at zero

Illya Karabash
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The talk is partially based on joint works with P. Binding, A. Kostenko, and M. Malamud.

Consider the equation $r(v)\psi_x(x,v) = \psi_{vv}(x,v) + f(x,v)$, $0 < x < 1$, $v \in \mathbb{R}$, and the associated half-range boundary value problem $\psi(0,v) = \varphi_+(v)$ if $v > 0$, $\psi(1,v) = \varphi_-(v)$ if $v < 0$. It is assumed that $vr(v) > 0$. So the weight function r changes its sign at 0. Boundary value problems of this type arise in a number of applications, e.g., in kinetic theory and electron scattering.

We consider the above equation in the abstract form $J\psi_x(x) + L\psi(x) = f(x)$, where J and L are operators in a Hilbert space H such that $J = J^* = J^{-1}$, $L = L^* \geq 0$. The case when L is nonnegative and has discrete spectrum or satisfies the weaker assumption $\inf \sigma_{ess}(L) > 0$ was described in great detail (see e.g. [1] and references therein). The latter assumption is not fulfilled for some models arising in the theory of stochastic processes (see [2]) and the complete existence and uniqueness theory for such cases have not been constructed yet. The reason is that the J-self-adjoint operator $A = JL$ may have a critical point at 0.

The existence and uniqueness theorem for the case of regular critical point 0 will be discussed. The second aim of the talk is to explain new methods and results concerning the regularity of the critical point 0.

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Wiener-Hopf operators with symbols generated by semi-almost periodic and slowly oscillating matrix functions on weighted Lebesgue spaces

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Given $1 < p < \infty$ and a Muckenhoupt weight $w \in A_p(\mathbf{R})$, let $M_{p,w}$ stand for the set of all Fourier multipliers on the weighted Lebesgue space $L^p(\mathbf{R}, w)$ equipped with the norm $\|f\|_{p,w} := \left(\int_{\mathbf{R}} |f(x)|^p w^p(x)dx\right)^{1/p}$. For Wiener-Hopf operators $W(a)$ with symbols a generated by semi-almost periodic and slowly oscillating matrix functions with entries in $M_{p,w}$, Fredholm criteria and index formulas are established on weighted Lebesgue spaces $L^p_N(\mathbf{R}_+, w)$ where $1 < p < \infty$, w belong to a subclass of Muckenhoupt weights and $N = 1, 2, \ldots$. We also study the invertibility of Wiener-Hopf operators with almost periodic matrix symbols on $L^p_N(\mathbf{R}_+, w)$. The talk is based on a joint work with Juan Loreto Hernández.

Classical orthogonal functions and semiseparable matrices

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Semiseparable matrices arise in a variety of problems. An interesting application is found in relation to fast transforms involving classical orthogonal polynomials and their associated functions.

Quite often, a key component of the respective algorithms is an efficient method to convert a finite expansion in one sort of orthogonal functions into one that uses a different family of functions. For example, an efficient method to convert an expansion in Legendre polynomials into one that uses Chebyshev polynomials of first kind, allows to use fast discrete cosine transforms to evaluate the expansion at a given set of nodes. Here, semiseparability appears in matrices whose eigenvectors are contained in the matrix representing the sought transformation between two families of orthogonal functions. In combination with well-known fast algorithms to apply these eigenvector matrices to a vector, the method becomes efficient.

We will survey results for classical orthogonal polynomials and their associated functions, and mention applications to fast Fourier transforms on the sphere $\mathbb{S}^2$, the rotation group $SO(n)$, and, as an outlook, generalizations thereof.

The Königs problem and extreme fixed points

Victor Khatskevich (presenter) and V. Senderov

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An affine f.l.m. $\mathcal{F}_A : \mathcal{K} \to \mathcal{K}$ of the unit operator-valued ball is considered in the case where the fixed point C of the continuation of $\mathcal{F}_A$ to $\overline{\mathcal{K}}$ is either an isometry or a coisometry. For the case in which one of the diagonal elements (for example, A_{11}) of the operator matrix A is identical, the structure of the other diagonal element (A_{22}) is studied completely. It is shown that, in all these reasonings, C cannot be replaced by an arbitrary point of the unit sphere; some special cases in which this is still possible are studied. In conclusion, the KE-property of the mapping $\mathcal{F}_A$ is proved.

CMV matrices and Adamjan-Arov scattering

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Adamjan-Arow scattering is applied to CMV matrices. It is shown that under Szegö assumption there exists a pair of $*$-cyclic wandering vectors such that Adamjan-Arov scattering function (with respect to this pair) coincides with the reflection coefficient. Direct and inverse scattering problems are considered for CMV matrices of Szegö class. Uniqueness of the inverse scattering is (roughly speaking) equivalent to Arov-regularity of the reflection coefficient. Joint work with F. Peherstorfer and P. Yuditskii (Johannes Kepler University, Austria).

Minimal normal and commuting completions

David P. Kimsey (presenter) and Hugo J. Woerdeman

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Given $A \in \mathbb{C}^{n\times n}$. Can we find a normal matrix $A_{ext} := \begin{pmatrix} A & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, where $A_{12} \in \mathbb{C}^{n\times k}$, $A_{21} \in \mathbb{C}^{k\times n}$, and $A_{22} \in \mathbb{C}^{k\times k}$, of smallest possible size? We shall call the smallest number of rows and columns we need to add the *normal defect* of A and denote by $\mathrm{nd}(A)$. As it turns out, in certain situations, it is useful to make A_{ext} to be a nonzero multiple of a unitary. The smallest possible k to do this shall be called the *unitary defect* of A. In this paper we provide a lower bound for $\mathrm{nd}(A)$ in terms of the inertia of the commutator $[A, A^*]$, that for certain matrices is sharp. For the case when $\mathrm{nd}(A) = 1$ we also investigate what freedom one has in constructing A_{ext}. In addition, we study the related question where $A \in \mathbb{C}^{n\times n}$ and $B \in \mathbb{C}^{n\times n}$ are given, and where we look for

$$A_{ext} := \begin{pmatrix} A & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad \text{and} \quad B_{ext} := \begin{pmatrix} B & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

such that they commute and are of smallest possible size.

Ascent and descent in multivariable spectral theory

Derek Kitson

School of Mathematics, Trinity College, Dublin 2, Ireland

In this talk the theory of ascent and descent for a linear operator acting on a vector space is extended to arbitrary sets of operators and applied to the study of joint spectra for finite commuting systems of bounded operators acting on a complex Banach space. To each member of a large class of joint spectra containing the Taylor spectrum, Harte spectrum, two-sided Slodkowski spectra and split versions we associate a Browder joint spectrum. We show that these Browder joint spectra are non-empty, compact-valued, have the projection property and consequently satisfy a spectral mapping theorem.

On the eigenvalues of Non-Hermitian Schrödinger operators

Martin Klaus

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We study the eigenvalues of certain Schrödinger operators with complex potentials. In particular, we are interested in conditions on the potential which guarantee that the spectrum is purely real. A typical class of Hamiltonians that is currently being discussed in the literature are the parity and time-reversal (PT) invariant Hamiltonians with supersymmetry. We employ a relationship between such Hamiltonians and Zakharov-Shabat (ZS) systems to conclude that if the ZS system has only purely imaginary eigenvalues, then the supersymmetric Hamiltonians has only real spectrum. In this context we will discuss a generalization of the "single-lobe" theorem to a wider class of potentials. We will also look at eigenvalue trajectories as the coupling constant varies and discuss some results on the location of resonances for these non-Hermitian operators.

Connes' embedding conjecture and sums of squares of NC polynomials

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We consider polynomials in NC variables. Which of these polynomials yield a matrix with positive trace whenever matrices are substituted for the variables? Can one find algebraic certificates implying this? Natural certificates involve sums of hermitian squares and can be searched for using semidefinite programming. We relate this to two open problems: Connes' embedding conjecture on type II_1 von Neumann algebras and the BMV Conjecture from statistical quantum mechanics. This is joint work with Markus Schweighofer.

Stable polynomials and distinguished varieties

Greg Knese

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The zero sets of two variable polynomials can interact in various ways with the two dimensional torus in $\mathbb{C}^2$. We will explore the relationship between polynomials with no zeros on the closed bidisk and distinguished varieties (plane curves that exit the bidisk through the distinguished boundary). As an application, we can reprove and provide some extra details to a known representation theorem for distinguished varieties. All of these topics are closely related to inner functions and Pick interpolation on the bidisk.

Revisit to a theorem of Wogen

Hyungwoon Koo

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In this note we reconstruct the proof of a theorem of Wogen on the boundedness criteria for a composition operator on $H^2(B^n)$ and apply to general maps. This proof can be extended to non-analytic symbols. As a result we provide a criteria for a composition operator to be bounded with non-analytic symbol.

On orthogonality of McDonald functions

Sherwin Kouchekian

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We will give a new orthogonality relation for McDonald functions via polynomial approximation which enters integral transforms such as those of Kontorovich-Lebedev and Mehler-Fock as kernels The orthogonality relation has important applications in boundary value problems of electrostatics and elasticity arising in spheroidal or cylindrical domains.

This result is a joint work with A. Passian, S. Yakubovich, and H. Simpson.

Constant-diagonal projections in finite-dimensional spaces

Leonid Kovalev (presenter) and David Larson

Department of Mathematics, Texas A&M University, College Station, TX 77843-3368

We show that the set of projections in $\mathbb{C}^{2n}$ with constant diagonal 1/2 is path-connected. The same is true for projections in $\mathbb{R}^{2n}$ as long as $n > 1$. This is joint work with Julien Giol, Nga Nguyen, and James Tener.

On almost periodic causal factorizations

Ilya Krishtal

Department of Mathematical Sciences, Northern Illinois University, DeKalb, IL 60115

In 1972 I. Gohberg and J. Leiterer proved a deep result which implies existence of causal factorizations of periodic operator-valued functions that are close to the identity. The main result of the talk is a similar theorem that pertains to an almost periodic setting. Examples include different factorizations of irregular time-frequency shifts and special classes of pseudo-differential operators. The talk is based on joint work with T. Strohmer.

Spectra of graph operators and thin structures

Peter Kuchment

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In the last couple of decades, many problems of mathematical physics, optics, microelectronics, and nanotechnology have led to studying spectra of differential operators in thin branching domains ("fattened graphs"). Since such a study poses great analytic and numerical difficulties, the natural idea has been pursued of approximating the true operators by those on one-dimensional structures (graphs). One of the main issues of interest has been approximations of spectra. The talk will provide a survey of the motivations and recent results of this research.

Spectral properties of totally positive operators in ideal spaces

Olga Kushel

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Let A be a completely continuous positive linear operator acting in a regular Banach ideal space $X(\Omega)$ under some natural conditions on Ω. In this case we can define operators $\otimes^j A$ and $\wedge^j A \quad (j = 1, 2, \ldots)$, i.e. the j-th tensor and the j-th exterior power of the operator A. The operator $\otimes^j A$ acts in the regular Banach ideal space $\otimes^j X = \widetilde{X_j}(\Omega^j)$, which is defined as the set of all functions $x(t_1, \ldots, t_j)$ on Ω^j, measurable (with respect to all the variables), for that the norm

$$\|x(t_1, \ldots, t_j)\|_{M_j} = \max_{\theta}\{\| \ldots \|x(t_1, \ldots, t_j)\|_{\theta(1)} \ldots \|_{\theta(j)}\}$$

is finite. Here $\theta = (\theta(1), \ldots, \theta(j))$ is a permutation of the set $(1, \ldots, j)$, and the indices $\theta(1), \ldots, \theta(j)$ mean, that the norm of the space X is taken firstly with respect to the variable $\theta(1)$, then to the variable $t_{\theta(2)}$ and so on to the $\theta(j)$-th variable. The operator $\wedge^j A$ acts in the space $\wedge^j X = \widetilde{X_j}^a(\Omega^j)$, which is a subspace of antisymmetric functions in $\otimes^j X = \widetilde{X_j}(\Omega^j)$. The space $\widetilde{X_j}^a(\Omega^j)$ is isomorphic to the space $\widetilde{X_j}(W_j)$, where W_j is a subset of Ω^j having some special properties.

Let the operator $R(\lambda, A) = (\lambda I - A)^{-1}$ be regular for every $\lambda \notin \sigma(A)$. Then the following equality holds:

$$\sigma_p(\otimes^j A) \setminus \{0\} = (\underbrace{\sigma_p(A)\ \ldots \sigma_p(A)}_{j}) \setminus \{0\}.$$

Here $\sigma_p(\otimes^j A)$ is the point spectrum of the operator $\otimes^j A$.

Let $\{\lambda_i\}$ be the set of all eigenvalues of the operator A, repeated according to multiplicity. Then all the possible products of the type $\{\lambda_{i_1} \ldots \lambda_{i_j}\}$, where $i_1 < \ldots < i_j$, form the set of all the possible (except, probably, zero) eigenvalues of the exterior power $\wedge^j A$, repeated according to multiplicity.

Let the operator A leave invariant the cone K of nonnegative functions in $X(\Omega)$, besides $\rho(A) > 0$, and there is only one eigenvalue on the spectral circle $|\lambda| = \rho(A)$. Let, for every j $(1 < j \leq k)$, the j-th exterior power $\wedge^j A$ leave invariant an almost reproducing cone K_j in $\widetilde{X_j}(W_j)$, besides $\rho(\wedge^j A) > 0$, and there is also only one eigenvalue on the spectral circle $|\lambda| = \rho(\wedge^j A)$. Then the operator A has k positive simple eigenvalues, that are different in modulus from each other and from the rest of eigenvalues : $0 < \lambda_k < \ldots < \lambda_2 < \lambda_1 = \rho(A)$.

Let the operator A (of a finite or infinite range r) leave invariant the cone K of nonnegative functions in $X(\Omega)$ and let A be indecomposable. Let, for every j $(j = 2, 3, \ldots, r)$, the j-th exterior power $\wedge^j A$ leave invariant an almost reproducing, miniedral and acute cone K_j in $\widetilde{X_j}(W_j)$, and let $\wedge^j A$ be also indecomposable. Let spectral radii $\rho(A)$ and $\rho(\wedge^j A)$ be poles of operators $R(\lambda, A)$ and $R(\lambda, \wedge^j A)$ respectively. Let $h(A)$ and $h(\wedge^j A)$ be the indices of imprimitivity of A and $\wedge^j A$ respectively. Then one of the following statements is true:

(a) $h(A) = 1$ and for every j the index $h(\wedge^j A) = 1$. In this case all the eigenvalues of A are positive, simple and different in modulus from each other.

(b) $h(A) = 3$, $h(\wedge^j A) = 1$ for j, which is divisible by 3, $h(\wedge^j A) = 3$ for j, which is not divisible by 3. In this case all the eigenvalues of A are simple, and for every $q \geq 0$ the eigenvalues $0 < \lambda_{3q+1} = |\lambda_{3q+2}| = |\lambda_{3q+3)}|$ coincide with the 3-th roots of λ_{3q+1}^3.

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Similarity of operators and the Contractive Assumption

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Looking at the eigenvector bundles of the operators to solve a problem in operator theory is the approach first taken by M. J. Cowen, R. G. Douglas and M. Uchiyama. With the assumption that only contractions be considered, one can characterize those operators that are similar to the backward shift on the Hardy class using their eigenvector structures (H. Kwon and S. Treil). We discuss the situation for noncontractive operators in this talk.

Quadratic eigenvalue problems, old and new

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The emphasis will be on the well-developed theory of Hermitian matrix polynomials. Some basic theory will be summarized concerning indefinite inner products, canonical triples, and sign characteristics and, thence, the fundamental factorization theorem. If time permits, the immediate implications for perturbation theory, and the role of the “moments” of the transfer function will be mentioned. The problem types: elliptic, mixed, and hyperbolic will be introduced and some recent results will be surveyed. I hope to present more recent results concerning inverse eigenvalue problems, eigenvalue/eigenvector updating, and reduction to diagonal forms. In large measure, this comprehensive theory was originated forty years ago and is due to the insight and perception of Israel Gohberg.

A note on the Fourier matrix

Peter Lancaster
Department of Mathematics, University of Calgary, Calgary, Alberta T2N 1N4, Canada

I will present a solution to a problem posed by a colleague concerning the Fourier matrix, namely: construct a smooth path of unitary matrices connecting the Fourier matrix to the identity. It is a nice exercise and tutorial in matrix analysis - one that IK might have appreciated.

Two-dimensional covariance matrices

Henry Landau
Bell Labs, Alcatel-Lucent

J. Geronimo and H. Woerdeman have characterized those $(mn) \times (mn)$ block Toeplitz matrices with Toeplitz blocks which are formed by trigonometric moments of a two-dimensional measure of the form $|P_{m,n}(z, w)|^{-2}$, $|z| = |w| = 1$, with $P_{m,n}(z, w)$ a polynomial of degrees m and n in z and w, respectively. We give another proof of this important result by applying ideas of prediction and the Gohberg-Semencul formula. We also show that the above measure generates the maximum-entropy extension of the given matrix. The talk is based on joint work with Israel Gohberg.

Fixed diagonal idempotents and dual frame pairs

David Larson (presenter) and Leonid Kovalev
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We show that the set of idempotent $n \times n$ matrices with constant diagonal $1/2$ is pathwise connected. As a consequence, this yields a connectedness result for dual frame pairs. We generalize our result to show that if D is the diagonal of some projection, then the set of all idempotents with diagonal D is pathwise connected. This project was motivated by the Kadison-Singer problem in the well-known equivalent form of the Paving problem for diagonal $1/2$ projections matrices.

The Gohberg theory of operators with semiseparable kernels, and the Evans function in stability of traveling waves

Yuri Latushkin
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We will discuss the recently discovered connections of the results on Fredholm determinants for integral operators with semiseparable kernels, originated in the classical work of I. Gohberg, S. Goldberg, M. A. Kaashoek, and N. Krupnik, and the Evans function, a Jost function type Wronskian which recently became one of the main instruments in stability theory of traveling waves.

In particular, generalizing a celebrated theorem of Jost and Pais, we will show how to express the Evans function via the (modified) Fredholm determinant of a Birman-Schwinger type operator, and how to compute the derivative of the Evans function at an eigenvalue.

This talk is based on the joint work with F. Gesztesy, K. A. Makarov and K. Zumbrun.

Cocycles in the theory of holomorphic operator functions

Jürgen Leiterer
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Israel Gohberg and I are preparing a book "Holomorphic Operator Functions of a Single Variable". This book will be focussed on the relations between local and global theories and is based on methods and technics of Complex analysis of several variables, for example the theory of cocycles of holomorphic functions with values in different groups (numbers, matrices, operators, sections of sheaves). The lecture will be an introduction to this subject. We start with the definition of a cocycle and the formulation of certain typical results. Then examples will be presented to show how this powerful theory works.

The sum of unitary similarity orbits contains only special operators

Chi-Kwong Li
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Let $\mathcal{B}(\mathcal{H})$ be the algebra of bounded linear operator acting on a Hilbert space $\mathcal{H}$ (over the complex or real field). Characterization is given to $A_1, \ldots, A_k \in \mathcal{B}(\mathcal{H})$ such that for any unitary operators $U_1, \ldots, U_k$, $\sum_{j=1}^{k} U_j^* A_j U_j$ is always in a special class $\mathcal{S}$ of operators such as normal operators, self-adjoint operators, unitary operators. As corollaries, characterizations are given to $A \in \mathcal{B}(\mathcal{H})$ such that the (real) linear or nonnegative combinations of operators in its unitary orbits $\mathcal{U}(A) = \{U^*AU : U \text{ unitary}\}$ always lie in $\mathcal{S}$.

The talk is based on a joint work with Yiu-Tung Poon.

On Burnside's Theorem on matrix algebras

Victor Lomonosov
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A simple proof of the classical Burnside's Theorem on matrix algebras will be presented with some generalization to infinite dimensional spaces.

On some categories and functors in the theory of quaternionic Bergman spaces

Maria Elena Luna-Elizarrarás

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There are presented basic facts about the theory of quaternionic Bergman spaces with special emphasis on what is happening with them under conformal transformations of the domains. There will be constructed a series of categories, whose objects are spaces of quaternion-valued functions, and we shall show that covariant functors acting between some of these categories, are related with some mathematical objects that belong to the theory of quaternionic Bergman spaces, characterizing their properties of being conformally covariant or invariant.

The talk is based on a joint work with J. O. González-Cervantes and M. Shapiro; it was partially supported by CONACYT projects as well as by Instituto Politécnico Nacional in the framework of COFAA and SIP programs.

The Gaussian semiclassical soliton ensemble for the focusing nonlinear Schrödinger equation

Gregory Lyng

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We are interested in the semiclassical ($\epsilon \downarrow 0$) limit of the initial-value problem for the focusing nonlinear Schrödinger equation,

$$\mathrm{i}\epsilon\psi_t + \frac{\epsilon^2}{2}\psi_{xx} + |\psi|^2\psi = 0\,, \quad \psi(x,0) = A_0(x)\,, \tag{3}$$

where A_0 is a positive real-valued function which decays rapidly as $|x| \to \infty$, and $\epsilon > 0$ is a parameter. The first step in solving (3) via the inverse-scattering transform is a spectral analysis of the nonselfadjoint Zakharov-Shabat eigenvalue problem with A_0 as the potential. It is known that if A_0 is a "single-lobe" potential, then the point spectrum is confined to the imaginary axis. Moreover, formal calculations reveal that, in the limit $\epsilon \downarrow 0$, the eigenvalue problem appears to go over to a semiclassically scaled Schrödinger operator plus a (supposedly) small correction. Applying the WKB method to this Schrödinger operator, we obtain, for each small $\epsilon > 0$, guesses for the discrete eigenvalues. Using these approximate eigenvalues (and other associated scattering data), it is then possible to construct an exact solution of the focusing nonlinear Schrödinger equation by inverse scattering. The collection of these solutions for values of ϵ tending to 0 is called a *semiclassical soliton ensemble*. The substitution of the approximate scattering data for the true scattering data associated with A_0 amounts to a modification of the initial data, and a basic question is the relationship of the solutions of the sequence of the modified initial-value problem to those of the true initial-value problem. In this case the challenge is to determine the relationship between the WKB and the true spectral data. In this talk, we shall discuss this problem and report on work in progress in the special case where $A_0(x) := \mathrm{e}^{-x^2}$.

A survey of some recent results on composition operators

Barbara MacCluer
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Given $\varphi : \Omega \to \Omega$, an analytic map of a domain Ω in $\mathbb{C}$ or $\mathbb{C}^n$, the composition operator C_φ is defined by $C_\varphi(f) = f \circ \varphi$ for f belonging to some Hilbert or Banach space of analytic functions on Ω. The study of composition operators affords a pleasing interplay between operator theory and complex analysis. In this talk, we give a "status report" on recent developments in selected areas of the field.

Structured mapping problems for matrices associated with an indefinite inner product

D. Steven Mackey
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Given a class of structured matrices $\mathbb{S}$, we identify pairs of vectors x, b for which there exists a matrix $A \in \mathbb{S}$ such that $Ax = b$, and also characterize the set of all matrices $A \in \mathbb{S}$ mapping x to b. The structured classes we consider are the self-adjoints, skew-adjoints, and isometries associated with an indefinite inner product. Some results on structured mappings with certain extremal properties will also be presented.

This is joint work with N. Mackey (Western Michigan University) and F. Tisseur (The University of Manchester).

On the definition of some natural classes of scalar products

Niloufer Mackey
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There are many useful properties of the standard inner product on $\mathbb{C}^n$ that are not shared by general scalar products. A number of these properties have proved to be essential in the investigation of canonical forms of structured matrices, structured factorizations, structure-preserving matrix iterations, and structured mappings. In this talk we consider a variety of such properties and show how they cluster together into equivalence classes, thus defining several natural classes of scalar products.

This is joint work with D. S. Mackey (Western Michigan University) and F. Tisseur (University of Manchester).

Meromorphic factorization and Toeplitz operators: some new relations

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Some properties and applications of meromorphic factorization of matrix functions are studied. It is shown that a meromorphic factorization of a matrix function G allows one to characterize the kernel of the Toeplitz operator with symbol G without actually having to previously obtain a Wiener-Hopf factorization. A method to turn a meromorphic factorization into a Wiener-Hopf one which avoids having to factorize a rational matrix that appears, in general, when each meromorphic factor is treated separately, is also presented. The results are applied to some classes of $n \times n$ matrix symbols.

This is joint work with C. Câmara.

An estimate for the number of solutions of an homogeneous generalized Riemann boundary value problem with shift

Rui Marreiros

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In the real space of all Lebesgue measurable complex valued functions on $\mathbb{R}$, which are absolutely integrable in the second power, $\widetilde{L}_2(\mathbb{R})$, we consider the generalized Riemann boundary value problem: find the functions $\varphi_+(z)$ and $\varphi_-(z)$ analytic in the upper halfplane and in the lower halfplane, respectively, satisfying the condition $\varphi_+ = a\varphi_- + b\varphi_-(\alpha) + c\overline{\varphi_-} + d\overline{\varphi_-(\alpha)}$, $\varphi_-(\infty) = 0$, imposed on their boundary values, i. e., on $\mathbb{R}$, where $\alpha(t) = t + h$, $h \in \mathbb{R}$, is the shift on the real line, and a, b, c, d are continuous functions on $\overset{\circ}{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$, the one point compactification of $\mathbb{R}$. Under certain conditions on the coefficients, an estimate for the number of linear independent solutions of this problem is obtained.

This is joint work with Viktor G. Kravchenko and Juan C. Rodriguez.

Hyperbolic geometry and norms of some composition operators on the Bloch space

María J. Martín

Department of Mathematics, University of Michigan, Ann Arbor, MI 48109

We derive an estimate and, for a large class of cases, an exact formula for the norm of a composition operator acting on the Bloch space in terms of the hyperbolic derivative and the hyperbolic distance. This is a joint work with D. Vukotić.

Integral representation formulas for Dirac and semi-Dirac pairs of differential operators

Mircea Martin

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The standard Euclidean Dirac operator $\mathfrak{D} = D_{\rm euc,n}$ is a first-order differential operator on $\mathbb{R}^n$, $n \geq 2$, with coefficients in the real Clifford algebra $\mathfrak{A}_n(\mathbb{R})$ associated with $\mathbb{R}^n$, that has the defining property

$$\mathfrak{D}\mathfrak{D}^\dagger = \mathfrak{D}^\dagger\mathfrak{D} = \Delta_{\rm euc,n},$$

where $\Delta_{\rm euc,n}$ stands for the Laplace operator on $\mathbb{R}^n$ and $\mathfrak{D}^\dagger = -\mathfrak{D}$. As natural extensions of this specific class of operators we are going to investigate pairs $(\mathfrak{D}, \mathfrak{D}^\dagger)$ of first-order homogeneous differential operators on $\mathbb{R}^n$ with coefficients in a real Banach algebra $\mathfrak{A}$, such that either

$$\mathfrak{D}\mathfrak{D}^\dagger = \mu_L\Delta_{\rm euc,n}, \quad \mathfrak{D}^\dagger\mathfrak{D} = \mu_R\Delta_{\rm euc,n},$$

or

$$\mathfrak{D}\mathfrak{D}^\dagger + \mathfrak{D}^\dagger\mathfrak{D} = \mu\Delta_{\rm euc,n},$$

where μ_L, μ_R, or μ are some elements of $\mathfrak{A}$. Every pair $(\mathfrak{D}, \mathfrak{D}^\dagger)$ that has the former property is called a *Dirac pair* of differential operators, and each pair $(\mathfrak{D}, \mathfrak{D}^\dagger)$ with the latter property is called a *semi-Dirac pair*. The simplest examples of a Dirac, or semi-Dirac pair of differential operators on $\mathbb{R}^n$ are given by $\mathfrak{D} = d + d^*$ and $\mathfrak{D}^\dagger = -(d + d^*)$, or $\mathfrak{D} = d$ and $\mathfrak{D}^\dagger = -d^*$, where d is the operator of exterior differentiation acting on smooth differential forms on $\mathbb{R}^n$, and d^* is its formal adjoint.

Our main goal is to prove that for any Dirac, or semi-Dirac, pair $(\mathfrak{D}, \mathfrak{D}^\dagger)$ we have two *Cauchy-Pompeiu type*, and two *Bochner-Martinelli-Koppelman type*, integral representation formulas in several real variables, respectively, one for $\mathfrak{D}$ and, as expected, another for $\mathfrak{D}^\dagger$. In addition, we will show that the existence of such integral representation formulas characterizes the two classes of pairs of differential operators.

Algebras of approximation sequences. Applications to spectra of convolution type operators on cones

Helena Mascarenhas

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We consider an algebra of operator sequences containing the approximation sequences to convolution type operators on cones acting on $L^p(\mathbb{R}^2)$, with $1 < p < \infty$. To each operator sequence (A_n) it is associated a set of operators $W_x(A_n) \in \mathcal{L}(L^p(\mathbb{R}^2))$. When (A_n) is a Fredholm sequence and $p = 2$, we know that the singular values of A_n have the α-splitting property with α being the sum of the kernel dimensions of $W_x(A_n)$. We prove, for $1 < p < \infty$, the analogous result by defining approximation numbers in the context of infinite dimensional Banach spaces and using them instead of singular values. We also show the convergence of the ϵ-pseudospectrum, norms of inverses and condition numbers for stable sequences. This is joint work with B. Silbermann.

Semi-algebraic geometry in a free $*$-algebra

Scott McCullough

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The talk will cover aspects of semi-algebraic geometry in the non-commutative setting of a free $*$-algebra. Because of connections to systems engineering and the theory of operator systems, issues of positivity and convexity for non-commutative polynomials and rational functions, and of convexity for non-commutative semi-algebraic sets will receive special attention.

Partial matrix convexity

Scott McCullough (presenter), Damon Hay, Bill Helton, and Adrian Lim

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Let a and x denote two classes of non-commuting variables which are formally self-adjoint. The structure of polynomials $p(a, x)$ which are some combination of convex or concave in one or both of the variables will be discussed.

Inverse scattering transform and matrix Zakharov-Shabat systems

Cornelis van der Mee (presenter) and F. Demontis

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Explicit Solutions of the cubic matrix nonlinear Schrödinger (mNLS) equation in the focusing case

$$i\frac{\partial q}{\partial t} + \frac{\partial^2 q}{\partial x^2} + 2qq^\dagger q = 0$$

are derived by using the inverse scattering transform (IST). This requires the time evolution of the scattering data of the matrix Zakharov-Shabat system

$$-iJ\frac{\partial \Psi}{\partial x}(x, \lambda; t) - V(x; t)\Psi(x, \lambda; t) = \lambda\Psi(x, \lambda; t), \qquad x \in \mathbb{R},$$

where

$$J = \begin{pmatrix} I_n & 0_{n\times m} \\ 0_{m\times n} & I_m \end{pmatrix}, \qquad V(x; t) = \begin{pmatrix} 0_{n\times n} & iq(x; t) \\ -iq(x; t)^\dagger & 0_{m\times m} \end{pmatrix},$$

as well as solving the Marchenko integral equation by separation of variables. The time evolution of the scattering data can be found by deriving the scattering matrix as the product of two wave operator matrices. The explicit solutions, which are global in position and time and decay exponentially as $x \to \pm\infty$, are also derived by direct substitution.

Singuluar-value-like decompositions for matrix triples

Christian Mehl

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The singular value decomposition is an important tool in Linear Algebra and Numerical Analysis. Besides providing a canonical form for a matrix A under unitary basis changes, it simultaneously displays the eigenvalues of the associated Hermitian matrices AA^* and A^*A. Similarly, one can ask the question if there is a canonical form for a complex matrix A that simultaneously displays canonical forms for the complex symmetric matrices AA^T and A^TA.

In this talk, we answer this question in a more general setting involving indefinite inner products and defining an analogue of the singular-value decomposition in real or complex indefinite inner product spaces.

Linearization of structured matrix polynomials

Christian Mehl

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The polynomial eigenvalue problem, i.e., the problem of finding $\lambda \in \mathbb{C}$ and $x \in \mathbb{C}^n$ such that

$$(\sum_{i=0}^{k} \lambda^i A_i)x = 0, \quad A_i \in \mathbb{C}^{n\times n}$$

is usually solved by linearization, i.e., by transforming the eigenvalue problem to a linear standard or generalized eigenvalue problem. The canonical way to do this is to use the well-known companion form. However, this approach has some disadvantages as the companion form usually does not reflect additional symmetries that may be present in the coefficient matrices A_i, i.e., in the case that all A_i are Hermitian, the companion form does not consist of Hermitian matrices.

In this talk, we discuss a systematic approach how to generate linearizations that automatically reflect additional structures of the underlying polynomial eigenvalue problem. This then allows the use of structure-preserving algorithms to solve the linearized eigenvalue problem.

Mittag-Leffler interpolation for rational matrix valued functions intertwining a pair of ODEs with a spectral parameter

Andrey Melnikov (presenter) and Victor Vinnikov

Department of Mathematics, Ben Gurion University, Beer Sheva 84105

We consider a class of rational in λ matrix valued functions $S(\lambda, t_2)$ that map solutions of one ODE with a spectral parameter λ

$$\lambda\sigma_2 u - \sigma_1 \frac{\partial}{\partial t_2} u + \gamma u = 0, \tag{1}$$

to solutions of another ODE with the spectral parameter λ:

$$\lambda\sigma_{2*}y - \sigma_{1*}\frac{\partial}{\partial t_2}y + \gamma_* y = 0. \tag{2}$$

Such functions naturally arise in the study of overdetermined $2D$ systems, invariant in one of the variables (say t_1). In this setting *transfer functions* of the $2D$ systems have the property of intertwining a pair of ODE's with a spectral parameter. These systems are interesting because the coefficients of the ODE's are "time"-varying (i.e. depend on the other variable, t_2).

A passage from input ODE to the output ODE is analogous to Backlund transformation and particularly for the Sturm-Liouville case it enables to pass from a simple differential equation (with zero potential) to much more complicated one.

It turns out that the poles (in λ) of a rational matrix valued function in our class do not depend on t_2. Furthermore, a pole triple of this function can be taken to be of the form $(C(t_2), A, B(t_2)\sigma_1)$ where A is a constant matrix, $C(t_2)$ is a matrix solution of the spectral problem (2) with matrix spectral parameter A and $B(t_2)\sigma_1$ is a solution of the "inverse input" spectral problem (1) with the spectral parameter A:

$$[\sigma_2^*\mu - \sigma_1^*\frac{d}{dt_2} - \gamma^* - \frac{d}{dt_2}\sigma_1^*]y_*(t_2) = 0. \tag{3}$$

Conversely, we solve a Mittag-Leffler type interpolation problem for functions in our class.

Theorem 1 *Suppose that we are given a pole triple $(C(t_2), A, B(t_2)\sigma_1)$ for each t_2 with constant A and $C(t_2), B(t_2)\sigma_1$ solutions of the output (2) and the inverse input (3) ODE's with the matrix spectral parameter A. Suppose also that an analytic function $D(t_2)$ (the value at infinity) is given. Then the matrix function*

$$S(\lambda, t_2) = D(t_2) + C(t_2)(\lambda I - A)^{-1}B(t_2)\sigma_1$$

maps solutions of (1) with spectral parameter λ to solutions of (2) with the same spectral parameter iff the linkage conditions are satisfied

$$\begin{aligned}&\sigma_{1*}^{-1}\sigma_{2*}D = D\sigma_1^{-1}\sigma_2,\\ &\sigma_{1*}D\sigma_1^{-1}\gamma = \sigma_{2*}C\widetilde{B}\sigma_1 - \sigma_{1*}C\widetilde{B}\sigma_2 - \sigma_{1*}\tfrac{d}{dt_2}D + \gamma_* D.\end{aligned}$$

This work generalizes classical result for rational matrix functions from [BGR].

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On the semiclassical limit for the Sine-Gordon equation

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The Cauchy initial-value problem for the sine-Gordon equation in laboratory coordinates arises in many physically relevant situations. For example, the idealized propagation of magnetic flux quanta in a long superconducting Josephson junction is described by this mathematical problem. Moreover, in this context, the disparity between laboratory scales and quantum scales suggests a semiclassical scaling in which a typical initial disturbance liberates an enormous number of flux quanta whose nonlinear interactions can become complicated. We will describe an ongoing research project to apply modern methods of asymptotic analysis for Riemann-Hilbert problems to calculate precise semiclassical asymptotics in this situation.

A discrete version of a theorem of Appell with applications to irrationality and transcendence questions in Number Theory

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In 1828 Clausen [2] published the fundamental result that describes the linear third order differential equation satisfied by the squares and the product of any two solutions of a linear second order differential equation in the real domain. The extension to the case of the cube of a solution of a second order equation dates to Appell [1] in 1880. In this talk we give a discrete analog of the Clausen-Appell theorem for a linear recurrence relation of arbitrary order. Then we apply the theory of disconjugate linear recurrence relations to the study of irrational quantities in number theory. In particular, for an irrational number associated with solutions of three-term linear recurrence relations we show that there exists a four-term linear recurrence relation whose solutions allow us to show that the number is a quadratic irrational if and only if the four-term recurrence relation has a principal solution of a certain type. The result is extended to higher order recurrence relations and a transcendence criterion can also be formulated in terms of these principal solutions.

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Sobolev estimates for the Green potential associated with the Robin-Laplacian in Lipschitz domains satisfying a uniform exterior ball condition

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We show that if $u = G_\lambda f$ is the solution operator for the Robin problem for the Laplacian, i.e. $\Delta u = f$ in Ω, $\partial_\nu u + \lambda u = 0$ on $\partial\Omega$ (with $0 \le \lambda \le \infty$), then $G_\lambda : L^p(\Omega) \to W^{2,p}(\Omega)$ is bounded if $1 < p \le 2$ and $\Omega \subset \mathbb{R}^n$ is a bounded Lipschitz domain satisfying a uniform exterior ball condition. This extends the earlier results of V. Adolfsson, B. Dahlberg, S. Fromm, D. Jerison, G. Verchota, and T. Wolff, who have dealt with Dirichlet ($\lambda = \infty$) and Neumann ($\lambda = 0$) boundary conditions.

Instability zones of Hill operators with singular H^{-1}-potentials

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The talk is based on recent joint results with Plamen Djakov (Sabanci University).

We extend our results [1] on the spectrum and the size of spectral gaps and spectral triangles of Hill operators with L^2–potentials and smoothness of the potential to the case of Hill operators with singular H^{-1}-potentials.

References

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Spectral synthesis and the lattice of certain weighted composition operators

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It is provided a complete description of the lattice of certain weighted composition operators on several spaces of analytic functions on the unit disk of the complex plane. This is achieved by using spectral synthesis.

Grothendieck inequalities—from classical to noncommutative

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In 1956 Grothendieck published his celebrated “Résumé”, containing a general theory of tensor norms on tensor products of Banach spaces, describing several operations to generate new norms from known ones, and studying the duality theory between these norms. Since 1968 it has had considerable influence on the development of Banach space theory. The highlight of the paper is the result now called Grothendieck's Theorem (inequality). The noncommutative version of Grothendieck's inequality (conjectured in the ”Résumé”) was first proved by Pisier under some approximability assumption, and obtained in full generality by Haagerup.

In 1991 Effros and Ruan conjectured that a certain Grothendieck-type inequality for a bilinear form on C*-algebras holds if (and only if) the bilinear form is jointly completely bounded. In 2002 Pisier and Shlyakhtenko proved that this inequality holds in the more general setting of operator spaces, provided that the operator spaces in question are exact. Moreover, they proved that the conjecture of Effros and Ruan holds for pairs of C*-algebras, of which at least one is exact. In recent joint work with Uffe Haagerup we prove that the Effros-Ruan conjecture holds for general C*-algebras, with constant one. More precisely, we show that for every jointly completely bounded (for short, j.c.b.) bilinear form on C*-algebras A and B, there exist states f_1, f_2 on A and g_1, g_2 on B such that for all $a \in A$ and $b \in B$,

$$|u(a,b)| \leq \|u\|_{\mathrm{jcb}} \left(f_1(aa^*)^{1/2} g_1(b^*b)^{1/2} + f_2(a^*a)^{1/2} g_2(bb^*)^{1/2} \right) .$$

While the approach by Pisier and Shlyakhtenko relies on free probability techniques, our proof uses more classical operator algebra theory, namely, Tomita-Takesaki theory and special properties of the Powers factors of type III_λ, $0 < \lambda < 1$.

An upper bound on the characteristic polynomial of a nonnegative matrix leading to a proof of the Boyle–Handelman conjecture

Michael Neumann (presenter) and Assaf Goldberger

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In their celebrated 1991 paper on the inverse eigenvalue problem for nonnegative matrices, Boyle and Handelman conjecture that if A is an $(n+1) \times (n+1)$ nonnegative matrix whose **nonzero** eigenvalues are: $\lambda_1 \geq |\lambda_i|$, $i = 2, \ldots, r+1$, $r \leq n$, then for all $x \geq \lambda_1$,

$$\prod_{i=1}^{r+1} (x - \lambda_i) \leq x^{r+1} - \lambda_1^{r+1}. \quad (*)$$

To date the status of this conjecture is that Ambikkumar and Drury (1997) showed that the conjecture is true when $2(r+1) \geq (n+1)$, while Koltracht, Neumann, and Xiao (1993) showed that the conjecture is true when $n \leq 4$ or when all the spectrum of A is real. They also showed that the conjecture is asymptotically true with the dimension.

Here we prove a slightly stronger inequality than in (*) from which it follows that the Boyle–Handelman conjecture is true. Actually, we do not start from the assumption that the λ_i's are eigenvalues of a nonnegative matrix, but that $\lambda_1, \ldots, \lambda_{r+1}$ satisfy that $\lambda_1 \geq |\lambda_i|$, $i = 2, \ldots, r+1$, and the trace conditions:

$$\sum_{i=1}^{r+1} \lambda_i^k \geq 0, \quad \text{for all } k \geq 1. \quad (**)$$

A strong form of the Boyle–Handelman conjectured in 2002 by the present authors says that (*) continues to hold if the trace inequalities in (**) hold only for $k = 1, \ldots, r+1$. We further improve here on earlier results of the authors concerning this stronger form of the Boyle–Handelman conjecture.

Lifted Linear Matrix Inequality (LMI) representation of convex sets

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This talk will present some new results on representing convex set by lifted LMI. For a given convex set $S = \{x \in R^n : g_1(x) \geq 0, \cdots, g_m(x) \geq 0\}$ defined by polynomials $g_i(x)$, it can be shown that S can be represented in form of lifted LMI when: (i) every g_i is sos-concave, i.e., $-\nabla 2g_i(x) = G_i(x)^T G_i(x)$ for some matrix polynomial $G_i(x)$. (ii) the boundary of S has positive curvature. This sufficient condition is not far away from being necessary.

This is joint work with Bill Helton.

Shapiro-Sundberg component problem for composition operators on the Hardy space

Pekka Nieminen

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We show that there are non-compact composition operators in the path component of compact ones on the Hardy space H^2 of the unit disc. This answers a question of Shapiro and Sundberg from 1990. As a main tool we use Aleksandrov-Clark measures. This is joint work with Eva Gallardo-Gutiérrez, María González and Eero Saksman.

Well and Ill Posed Inversion Problems

Nikolai K. Nikolski
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We study possible discrepancies between the condition number $CN(T) = \|T\| \cdot \|T^{-1}\|$ and the spectral condition number $SCN(T) = \|T\| r(T^{-1})$ for large (nonselfadjoint) matrices or operators T, $r(\cdot)$ being the spectral radius (so that $r(T^{-1}) = 1/min_j|\lambda_j(T)|$ for the matrix case, $\lambda_j(T)$ are eigenvalues). Given a family $\mathcal{F}$ of matrices/operators we say that the inversion problem in $\mathcal{F}$ is well posed ("there are no hidden parameters") if there exists a function φ such that $CN(T) \leq \varphi(SCN(T))$ for every $T \in \mathcal{F}$. The growth rate of $\varphi(\delta)$ as $\delta \longrightarrow 0+$ describes the complexity of the inversion problem in $\mathcal{F}$ with respect to the selfadjoint case.

We examine the well/ill posedness of the inversion problem for several useful classes $\mathcal{F}$, such as 1) all $n \times n$ matrices with respect to the euclidean norm (Kronecker), 2) all $n \times n$ matrices with respect to an arbitrary norm (Van der Waerden's problem), 3) operators/matrices having a prescribed functional calculus/growth rate of the resolvent, 4) algebras of matrices/operators with a given spectrum.

Multipliers of Muckenhoupt bases, and the bounded inversion conjecture

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Let w be a weight function on the torus $\mathbb{T}$ satisfying the Muckenhoupt (A_2) condition. We discuss possibilities to use the Hadamard multipliers of the basis of exponentials $(e^{inx})_{n\in\mathbb{Z}}$ in $L^2(\mathbb{T}, w)$ in order to find a counterexample to the Bourgain-Tzafriri's bounded inversion problem. As is known, the latter one is equivalent to the Kadison-Singer problem.

Can one invert a matrix via graph manipulations?

Vadim Olshevsky
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We use signal flow graphs to describe the structure of the inverse polynomial Vandermonde matrix (and to design a fast $O(n2)$ algorithm that computes it). Although all the results can be derived algebraically, here we reveal a connection to signal processing, and deduce new inversion formulas via elementary operations on signal flow graphs for digital filter structures. We introduce, for the first time, several new filter structures (e.g., quasiseparable filters, semiseparable filters, well-free filters) that generalize the celebrated Markel-Grey filter widely used in speech processing. No knowledge of system theory and signal processing (or anything beyond matrices) is required, we will start with an elementary 5-minutes tutorial on flow graphs, and show how their use dramatically simplifies the derivation of inversion formulas. This is joint work with Tom Bella and Pavel Zhlobich.

Nuclearity of Hankel operators: applications in control theory

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Whether an abstract controlled differential equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned}$$

on a Hilbert space X can be approximated (in a for control relevant metric) by a system of the same type but defined on a finite dimensional space is closely related to properties of the Hankel operator of the system. This Hankel operator has symbol

$$G(s) := C(sI - A)^{-1}B + D,$$

the transfer function of the system. The for control relevant convergence is convergence of the transfer function in the Hardy space H^∞ of the right half-plane. A necessary condition for the existence of convergent approximations is that the Hankel operator is compact, a sufficient condition is that the Hankel operator is nuclear (trace class). In PDE examples the transfer function is usually not explicitly available or has a complicated structure. Therefore known characterizations of compactness and nuclearity in terms of the symbol often cannot be used. In this talk we give conditions for compactness and nuclearity in terms of the state space parameters A, B, C, D and use these conditions to show that some controlled PDEs can be approximated in the H^∞ norm by finite-dimensional systems and some can not.

Additive preconditioning in matrix computations

Victor Pan

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Compared to the customary techniques, we generate preconditioners more readily and for a much wider class of input matrices. We more readily preserve matrix structure and sparseness and develop a wider range of applications, in particular to linear systems with rectangular coefficient matrices and to eigen-solving. Our analysis and experiments show the power of our approach.

The distance from a matrix polynomial to matrix polynomials with a prescribed multiple eigenvalue, and condition numbers of eigenvalues

Nikolaos Papathanasiou

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In the first part of this presentation, we introduce a (spectral norm) distance from an $n \times n$ matrix polynomial $P(\lambda)$ to the matrix polynomials that have a given complex number μ,

(i) as an eigenvalue of geometric multiplicity at least κ, or

(ii) as a multiple eigenvalue.

Then we compute the first distance and obtain bounds for the second one, constructing associated perturbations of $P(\lambda)$.

In the second part, we investigate condition numbers of eigenvalue problems of matrix polynomials, generalizing classical results of matrix perturbation theory. We obtain that a matrix polynomial that has an ill-conditioned eigenproblem in some respects, it is also close to a matrix polynomial with multiple eigenvalues, and we construct an upper bound for this distance. Finally, a Bauer-Fike type theorem and an Elsner type theorem for matrix polynomials are proposed.

This is joint work with Panayiotis Psarrakos.

Indefinite Sturm-Liouville operators

Friedrich Philipp

Technische Universität Berlin, Berlin 10623, Germany

We consider the Sturm-Liouville differential expressions

$$\ell f = \frac{1}{w}\left((-pf')' + qf\right) \quad \text{and} \quad \tau f = \frac{1}{|w|}\left((-pf')' + qf\right)$$

with an indefinite weight w on $\mathbb{R}$, where w, q and $1/p$ are assumed to be real-valued and locally integrable, $p > 0$ and $w \neq 0$ a.e. Furthermore we suppose the following:

($*$) The differential expression τ is in the limit point case at $\pm\infty$.

By $L^2_{|w|}(\mathbb{R})$ we denote the Hilbert space of (equivalence classes of) all functions $f : \mathbb{R} \to \mathbb{C}$ such that $|f|^2|w|$ is integrable over $\mathbb{R}$. Furthermore, the inner product space $(L^2_{|w|}(\mathbb{R}), [\cdot,\cdot])$ where

$$[f,g] := \int_{\mathbb{R}} f(x)\,\overline{g(x)}w(x)\,\mathrm{d}x\,, \quad f,g \in L^2_{|w|}(\mathbb{R}),$$

will be denoted by $L^2_w(\mathbb{R})$. The maximal domain of ℓ and τ in $L^2_{|w|}(\mathbb{R})$ is defined as follows:

$$\mathcal{D}_{\max} := \left\{ f \in L^2_{|w|}(\mathbb{R}) : f, pf' \text{ absolutely continuous and } \tau f \in L^2_{|w|}(\mathbb{R}) \right\}.$$

On $\mathcal{D}_{\max}$ we define the operators A and B by $Af := \ell f$ and $Bf := \tau f$, $f \in \mathcal{D}_{\max}$. It is well-known that the operator A is selfadjoint in the Krein space $L^2_w(\mathbb{R})$ and that B is selfadjoint in the Hilbert space $L^2_{|w|}(\mathbb{R})$.

In this talk we will outline the known spectral properties of the operator A. For example, it is still an open question if the resolvent set of the operator A is always non-empty. We will give some conditions which imply $\rho(A) \neq \emptyset$.

The case $p \equiv 1$ and $w(x) = \operatorname{sign}(x)$ is well studied in the literature, e.g., it is well-known that $\rho(A) \neq \emptyset$ and that the non-real spectrum of A is discrete. We suppose furthermore that the limits

$$q_{-\infty} := \lim_{x\to-\infty} q(x) \quad \text{and} \quad q_{+\infty} := \lim_{x\to+\infty} q(x)$$

exist which in particular implies ($*$). Then, $\sigma_{\mathrm{ess}}(A) \subset (-\infty, -q_{-\infty}] \cup [q_{+\infty}, \infty)$ and it can be shown that the operator A is definitizable over the set $(-\infty, q_{+\infty}) \cup (-q_{-\infty}, \infty)$. In particular, the non-real spectrum of A can only accumulate to points contained in the (real) complement of this set. We give some numerical examples.

The talk is based on joint work with Jussi Behrndt (Berlin, Germany) and Carsten Trunk (Ilmenau, Germany).

Composition operators and complex interpolation

Matthew A. Pons

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Spectral results for composition operators with automorphic symbol acting on the Hardy space of the disk have been known since the late 60's and these were later generalized to the weighted Bergman spaces and most recently to the Dirichlet space. We first consider extending these results to the weighted Dirichlet spaces of the unit disk, which include the spaces listed above, and discuss the difficulties in using the known methods to gain more information. We employ the method of complex interpolation due to A. P. Calderón to deal with the spaces in a unified manner after obtaining a general result for identifying the spectrum of an operator acting on a family of interpolation spaces. Extensions to higher dimensions and other families of spaces are also explored.

The joint essential numerical range of operators: convexity and related results

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Denote by $W(\mathbf{A})$ and $W_e(\mathbf{A})$ the joint numerical range and the essential joint numerical range of an m-tuple of self-adjoint operators $\mathbf{A} = (A_1, \ldots, A_m)$ acting on an infinite dimensional Hilbert space. While $W(\mathbf{A})$ is not convex in general, it is shown that $W_e(\mathbf{A})$ is always convex and the closure of $W(\mathbf{A})$ is always star-shaped with the elements in $W_e(\mathbf{A})$ as star centers. Although $\mathbf{cl}(W(\mathbf{A}))$ is usually not convex, an analog of the separation theorem is obtained, namely, for any element $\mathbf{d} \notin W(\mathbf{A})$, there is a linear functional f such that $f(\mathbf{d}) \geq \sup\{f(\mathbf{a}) : \mathbf{a} \in W(\tilde{\mathbf{A}})\}$, where $\tilde{\mathbf{A}}$ is obtained from $\mathbf{A}$ by perturbing one of the components A_j by a finite rank self-adjoint operator. To prove the main results, several equivalent definitions of $W_e(\mathbf{A})$ are established. Using the convexity of $W_e(\mathbf{A})$ one can derive additional equivalent definitions of $W_e(\mathbf{A})$ in terms of linear combinations of the operators $A_1, \ldots, A_m$. Consequently, some joint behaviors of $A_1, \ldots, A_m$ can be understood via their linear combinations. Other results concerning the perturbation of $A_1, \ldots, A_m$ by finite rank or compact operators are obtained.

The talk is based on a joint work with Chi-Kwong Li.

Operator theory on noncommutative domains

Gelu Popescu

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We study noncommutative domains $D_f \subset B(H)^n$ generated by positive regular free holomorphic functions f, where $B(H)$ is the algebra of all bounded linear operators on a Hilbert space H.

Each such a domain has a universal model $(W_1, \ldots, W_n)$ of weighted shifts acting on the full Fock space with n generators. The study of D_f is close related to the study of the the weighted shifts $W_1, \ldots, W_n$, their joint invariant subspaces, and the representations of the algebras they generate: the domain algebra $A_n(D_f)$, the Hardy algebra $F_n^\infty(D_f)$, and the C^*-algebra $C^*(W_1, \ldots, W_n)$. A good part of the talk deals with these issues. We discuss problems related to the dilation theory, model theory, and unitary invariants on noncommutative domains and noncommutative varieties. Commutant lifting results and applications are also considered.

Normal matrix polynomials

Panayiotis Psarrakos

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In this paper, we introduce the notions of weakly normal and normal matrix polynomials, with nonsingular leading coefficients. We characterize these matrix polynomials, using orthonormal systems of eigenvectors and normal eigenvalues. We also study the conditioning of the eigenvalue problem of a normal matrix polynomial, constructing an appropriate Jordan canonical form. Moreover, we introduce a (weighted) distance from normality for matrix polynomials, and study its relation with pseudospectra.

Algebraic theory of signal transform algorithms

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Linear transforms such as the discrete Fourier transform (DFT), the discrete cosine and sine transforms, and others, are widely used in digital signal processing. Consequently, there is a large body of literature on fast transform algorithms. With few exceptions these algorithms are derived by clever and often lengthy manipulation of the transform coefficients. These derivations are hard to grasp, and provide little insight into the structure of the resulting algorithm. Further, it is hard to determine if all relevant classes of algorithms have been found. This is not just an academic problem as the variety of different implementation platforms and application requirements makes a thorough knowledge of the algorithm space crucial. For example, state-of-the-art implementations of the DFT heavily rely on the existence of algorithm variants to adapt the implementation to the memory hierarchy or to optimize it for vector instructions and multiple threads.

In this presentation we show how to derive transform algorithms *algebraically.* This means that we do not manipulate the actual transform to obtain an algorithm, but decompose a *polynomial algebra* associated with the transform. We show two general decomposition theorems for polynomial algebras that 1) can explain most of the algorithms in the literature, 2) greatly simplify the algorithm derivation, and 3) produce novel, relevant algorithms that had not been found with previous methods.

Non-relatively compact perturbations and related topics

Jian-Gang Qi (presenter) and Shaozhu Chen
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Weyl's Spectral Theorem says that the essential spectrum of a self-adjoint operator is invariant under relatively compact symmetric perturbations of the operator. We present a class of perturbations that preserve the essential spectrum, but are not necessarily relatively compact. Two examples of such non-relatively compact perturbations are given, and one of them is the local dilative perturbation. The new concept and techniques can also be applied to spectral problems for partial differential operators.

In order to prove the invariance of the essential spectrum, we establish some new results in perturbation theory of operators. One result is about the equivalence of relative boundedness (compactness) of the minimal operator and the maximal operator when the deficiency indices are finite; and another one is about the equivalence of relative boundedness with the relative bound being zero between relative compactness with respect to a resolvent compact operator.

Logmodularity and the Kadison-Singer problem

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Logmodular algebras and the Kadison-Singer problem are related to the problem of unique extensions. In this talk we will show how Hoffman's ideas on logmodularity can be used to give a proof that paving the upper triangulars is equivalent to Anderson's original paving conjecture. This is joint work with Vern I. Paulsen.

Almost periodic factorization of matrix functions with Bohr-Fourier spectrum in a two-generator grid

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Conditions for almost periodic (AP) factorability of a 2×2 triangular matrix function with a particular quadrinomial in the off-diagonal entry were recently obtained by Avdonin, et al. In this talk, we introduce various new matrix functions whose AP factorability can be determined from this result by application of the Portuguese Transformation. In these new matrix functions, the off-diagonal entry is an AP polynomial whose Bohr-Fourier spectrum is contained in some two-generator grid. We also present a new class of matrix functions with trinomials in the off-diagonal entry, whose factorability can be obtained from these results. This is joint work with L. Rodman and I. Spitkovsky.

Application of a spectral expansion method to problems in physics

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The more than 10 year long collaboration between Israel Koltracht (mathematician) and the present author resulted in a new formulation of a spectral expansion method in terms of Chebyshev polynomials appropriate for solving a Lippmann-Schwinger integral equation in one dimension. Applications range from the solution of the two-body Schroedinger equation for either scattering or bound state conditions in atomic physics, to calculating the propagation of waves on an inhomogeneous string or on a bridge (in progress), and to solving the quantum mechanical three-body Faddeev integral equations in configuration space (in progress). These applications easily achieve an accuracy of better than eight significant figures, and were inspired by the creativity and untiring guidance of Professor I. Koltracht.

The absolutely continuous spectrum and reflectionless Jacobi matrices

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We are interested in general structural properties of Jacobi operators with some absolutely continuous spectrum. It turns out that there are universal building blocks of a very special type that have to be used to produce any kind of absolutely continuous spectrum. More precisely, this role is played by the reflectionless operators mentioned in the title (I'll recall the definition in the talk). I would like to discuss this result and its ramifications and also the more recent attempts to systematically analyze reflectionless operators by A. Poltoratski and myself.

If interested, please also see `http://arxiv.org/abs/0706.1101` for further reading.

Reflectionless Herglotz functions and Jacobi matrices

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Reflectionless Jacobi matrices seem to deserve detailed study because they may be viewed as the fundamental building blocks of general operators with some absolutely continuous spectrum. I'll review this result, including the definition of reflectionless, and then report on some recent attempts in this direction by A. Poltoratski and myself.

A quantitative estimate for bounded point evaluations in $P^t(\mu)$-spaces

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By use of X. Tolsa's work on analytic capacity we obtain a quantitative version of J. Thomson's theorem on bounded point evaluations for $P^t(\mu)$-spaces. A similar result can also be derived from a recent paper by J. Brennan, whose work also relies on Tolsa's Theorem.

In this talk I will discuss these results and also indicate how the basic lemma can be used to prove the existence of nontangential limits of functions in analytic $P^t(\mu)$-spaces on the unit disc, if μ has a nontrivial part on the unit circle.

This is joint work with Alexandru Aleman and Carl Sundberg.

The Fredholm index of band-dominated operators

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In 2004, V. S. Rabinovich, J. Roe and the author derived a formula which expresses the Fredholm index of a band-dominated operator on $l^p(\mathbb{Z})$ in terms of local indices of its limit operators. The proof makes thorough use of K-theory for C^*-algebras which, of course, appears as a natural approach to index problems.

The purpose of this talk is to develop a completely different approach to the index formula for band-dominated operators which is exclusively based on ideas and results from asymptotic numerical analysis. The main tools are a Fredholm theory for approximation sequences and an identity for the so-called α-number of a Fredholm sequence, which can be considered as an analogue of the kernel dimension of a Fredholm operator.

These results are a joint work with V. S. Rabinovich and B. Silbermann.

Toeplitz-plus-Hankel Bezoutians and inverses of Toeplitz and Toeplitz-plus-Hankel matrices

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Bezoutian-type formulas for the inverses of Toeplitz-plus-Hankel ($T+H$) matrices are presented which involve bases of kernels of associated rectangular T+H matrices. Special Bezoutians of this type yield inverses of symmetric or skewsymmetric Toeplitz matrices and vice versa. In both cases these representations lead to splitting formulas for inverses of centrosymmetric or centro-skewsymmetric $T + H$ matrices.

Generalized difference kernels having a finite number of negative squares

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This talk is based on joint work with L. A. Sakhnovich. Let $s(x) = -s^*(-x)$ be an $m \times m$ matrix-valued function on $(-\ell, \ell)$ such that the operator

$$(Sf)(x) = \frac{d}{dx} \int_0^\ell s(x-t) f(t)\, dt$$

is everywhere defined and bounded on $L^2_m(0, \ell)$. We then call $s(x-t)$ a **Hermitian generalized difference kernel.** We say that the kernel has a finite number of negative squares if the negative spectrum of S consists of eigenvalues of finite total multiplicity. Operator identities and a continual interpolation theorem are used to derive a representation for Hermitian generalized difference kernels which have a finite number of negative squares. The elements of the representation are taken from the Kreĭn-Langer integral representation of a generalized Nevanlinna function. The nondegenerate case is assumed. Such representations provide extensions of kernels to larger intervals with a bound on the number of negative squares.

On conservation laws for the KdV equation with rough initial data

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We introduce a sequence of conservation laws of the Faddeev-Zhakharov type which remains valid for rough solutions of the KdV equation on the full. Some applications to the spectral theory of one-dimensional Schrodinger operators with singular potentials will also be discussed.

On the solutions of Knizhnik-Zamolodchikov system

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The Knizhnik-Zamolodchikov system have found applications in several areas of mathematics and physics. We prove the following assertion: *The fundamental solution of KZ system is rational when k is integer.* We give an effective method of constructing this rational solution. Our method is elementary and is based on the results of linear algebra. More complicated approach to constructing rational solutions is given by G. Felder and A. Veselov. In our previous papers we constructed rational solutions only for the cases $k = 1$ and $k = -1$. We deduce the asymptotic formula for solutions of KZ system when k is integer. The corresponding result for irrational k is well known. We consider the examples $n = 3$ and $n = 4$. As a byproduct we obtain the following result: The hypergeometric function $F(a, b, c)$ is rational when $a = -k$, $b = -3k$, $c = 1 - 2k$ (k is integer).

The Hermite property of a causal Wiener algebra used in control theory

Amol Sasane

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Let $\mathbb{C}_+ := \{s \in \mathbb{C} \mid \mathrm{Re}(s) \geq 0\}$ and let $\mathcal{A}$ denote the Banach algebra

$$\mathcal{A} = \left\{ s(\in \mathbb{C}_+) \mapsto \widehat{f_a}(s) + \sum_{k=0}^{\infty} f_k e^{-st_k} \;\middle|\; \begin{array}{l} f_a \in L^1(0,\infty),\ (f_k)_{k\geq 0} \in \ell^1, \\ 0 = t_0 < t_1 < t_2 < \dots \end{array} \right\}$$

equipped with pointwise operations and the norm:

$$\|f\| = \|f_a\|_{L^1} + \|(f_k)_{k\geq 0}\|_{\ell^1}, \quad f(s) = \widehat{f_a}(s) + \sum_{k=0}^{\infty} f_k e^{-st_k} \quad (s \in \mathbb{C}_+).$$

(Here $\widehat{f_a}$ denotes the Laplace transform of f_a.) It is shown that, endowed with the Gelfand topology, the maximal ideal space of $\mathcal{A}$ is contractible.

In particular, it follows from the contractibility of $M(\mathcal{A})$ that the ring $\mathcal{A}$ is Hermite. The algebra $\mathcal{A}$ arises in control theory, and the Hermite property has useful consequences in the problem of stabilization of linear systems. Indeed, $\mathcal{A}$ being Hermite implies that if a transfer function G has a right (or left) coprime factorization, then G has a doubly coprime factorization, and the standard Youla parameterization yields all stabilizing controllers for G.

Using the (known) corona theorem for $\mathcal{A}$, and in light of the above Hermite property of $\mathcal{A}$, the following statements are equivalent for $f \in \mathcal{A}^{n\times k}$, $k < n$:

(1) There exists a $g \in \mathcal{A}^{k\times n}$ such that $gf = I_k$ on $\mathbb{C}_+$.
(2) There exist $F, G \in \mathcal{A}^{n\times n}$ such that $GF = I_n$ on $\mathbb{C}_+$ and $F_{ij} = f_{ij}$, $1 \leq i \leq n$, $1 \leq j \leq k$.
(3) There exists a $\delta > 0$ such that $f(s)^* f(s) \geq \delta^2 I_k$, $s \in \mathbb{C}_+$.

Max balancing matrices

Hans Schneider

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In this talk I define max balanced matrices and show that each irreducible nonnegative matrix is diagonally similar to a unique max balanced matrix. The ideas behind the algorithmic proof of this theorem are discussed. This part of the talk is based on joint work with M.H. Schneider and U.G.Rothblum done almost 20 years ago. It may be linked to as yet incomplete work on max algebras.

Least-squares approximations by elements from matrix orbits

Thomas Schulte-Herbruggen

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Let (A) denote the orbit of a complex or real matrix A under a certain equivalence relation such as unitary similarity, unitary equivalence, etc. Based on the differential geometry of the various orbits seen as Riemannian manifolds with bi-invariant metrics on their respective tangent spaces, efficient gradient-flow algorithms are constructed to determine the best approximation of a given matrix A_0 by the sum of matrices in $S(A_1), \ldots, S(A_N)$ in the sense of finding the Euclidean least-squares distance

$$\min\{\|X_1 + \cdots + X_N - A_0\| : X_j \in S(A_j),\ j = 1, \ldots, N\}.$$

Interrelations of the results to different pure and applied areas are discussed with special focus on applications in quantum dynamics of open systems.

Banach algebras of structured matrix sequences

Markus Seidel

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The asymptotic behavior of matrix sequences has been considered for decades and there are many important results on the stability and more generally on the Fredholm property of such sequences, in particular for finite sections of Toeplitz and band-dominated operators as well as for several discretization methods which were applied to singular integral operators. A basic idea is to introduce a suitable Banach algebra containing the sequences under consideration, and furthermore a family of homomorphisms which provide "snapshots" of the elements of the Banach algebra. Then certain properties of the sequences can be derived from their snapshots. More precisely, for a sequence $\{A_n\}$ each snapshot $W^t\{A_n\}$ is the strong limit of a modification of the original sequence, and its Fredholm properties turn out to be closely related to the Fredholm properties of $\{A_n\}$.

The most complete results concerning stability and the asymptotic behavior of singular values are available for C*-algebras (see for instance [2]). For the case of Banach algebras see [1] or [3]. Actually all of these results utilize the strong convergence to the operators $W^t\{A_n\}$ and therefore they are not applicable to finite section sequences of operators acting on l^∞ spaces, for example.

The purpose of this talk is to present a uniform approach which also coveres suitable non-strongly converging sequences, and to deduce analogous results. Moreover we will apply these results to band-dominated operators on $l^p(\mathbb{Z}, \mathbb{C}^N)$ and their finite section sequences for $1 \leq p \leq \infty$. This is joint work with B. Silbermann.

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On relations between some operators of quaternionic analysis and their counterparts in Clifford analysis

Michael Shapiro (presenter) and Juan Bory Reyes
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Let n be a positive integer, denote by $Cl_{0,n}$ the universal Clifford algebra of signature $(0,n)$ with the imaginary units $e_1, \dots e_n$. The operator $\mathcal{D}_n := \sum_{k=0}^{n} e_k \frac{\partial}{\partial x_k}$ is frequently called the (Cliffordian) Cauchy-Riemann operator while the operator $D_n := \sum_{k=1}^{n} e_k \frac{\partial}{\partial x_k}$ is usually termed the Dirac operator. Thus the two versions of Clifford analysis have arisen, one related to the Cauchy-Riemann operator, and other related to the Dirac operator. In case of $Cl_{0,2} \approx \mathbb{H}$, the skew-field of quaternions, there are defined the operator $\mathcal{D}_F := \frac{\partial}{\partial x_0} + e_1 \frac{\partial}{\partial x_1} + e_2 \frac{\partial}{\partial x_2} + e_{12} \frac{\partial}{\partial x_3}$, called the Fueter operator, and the operator $D_{MT} := e_1 \frac{\partial}{\partial x_1} + e_2 \frac{\partial}{\partial x_2} + e_{12} \frac{\partial}{\partial x_3}$, called the Moisil-Théodoresco operator; historically the function theories for them are referred to as quaternionic analysis.

The aim of the talk is to show that there does exist a precise relationship between both versions of quaternionic analysis and Clifford analysis for the algebra $Cl_{0,3}$ thus refining what has been established in [1].

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Spectral theory of Sturm-Liouville difference operators

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In this talk, we present answers to three basic questions about discrete Sturm-Liouville problems (DSLPs). First, how to tell whether a given DSLP can be realized as the spectral problem for a difference operator? Second, how to construct such a difference operator when it is known that there is one? Third, how to determine if such a difference operator is self-adjoint?

A main result of this paper is a general procedure for constructing (the domain $\mathcal{D}$ of) the associated difference operator when it exists. We would like to emphasize that the general procedure applies to all DSLPs: given a DSLP, if $\mathcal{D}$ exists, the procedure yields it, no matter whether the DSLP is self-adjoint or not; and if $\mathcal{D}$ does not exist, the procedure tells so.

Moreover, using the general procedure, we prove that a DSLP admits a difference operator realization if and only if it does not have all complex numbers as eigenvalues.

Thirdly, guided by the general procedure, we obtain explicit DSLPs with a definite weight function, but having all complex numbers as eigenvalues.

When $\mathcal{D}$ exists, in general, it is smaller than the space of all functions satisfying the boundary condition in the DSLP; and we give the minimum conditions for the difference operator to be self-adjoint with respect to a natural quadratic form.

These minimum conditions allow us to construct several classes of explicit *self-adjoint* Sturm-Liouville difference operators with either a *non-Hermitian* leading coefficient function, or a *non-Hermitian* potential function, or a *non-definite* weight function, or a *non-self-adjoint* boundary condition.

To the best of our knowledge, we believe that in the theory of continuous Sturm-Liouville problems, no such example is known.

Spectral properties of self-adjoint Sturm-Liouville difference operators are studied. In particular, several eigenvalue comparison results are proved.

This is joint work with Hongyou Wu.

Asymptotic and spectral analysis of aircraft wing model in subsonic airflow. Applications to flutter control

Marianna Shubov

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The objective of this talk is to present results obtained by the speaker in a series of papers. The works are devoted to detailed asymptotic and spectral analysis of a model of a long slender aircraft wing in a subsonic, incompressible, inviscid airflow.

The model has been developed in the Flight Systems Research Center of University of Los Angeles (California) in collaboration with NASA Dryden Center and has been tested in actual flight conditions at Edwards Air–force Base (California). The main goal of the entire project is to develop a strategy for a flutter control. NASA Aeronautics treats "Flutter Project" as its one of the most important research projects. The model to be presented is the only known linear model capable to capture flutter in long flexible moving solid structure (a wing).

The model is governed by a system of two coupled integro-differential equations. The differential part of the model (the structural part) is represented by a system of two coupled hyperbolic equations, which are the equations of motion of coupled Euler-Bernoulli and Timoshenko beam models. The differential part governs the ground vibrations of the wing (the bending–torsion vibration model), i.e., vibrations in the absence of a surrounding airflow. The integral part of the system (the aerodynamical part) consists of two complicated time-convolution type integrals. This part models forces and moments exerted on the wing by the airflow. The system is supplied with a two-parameter family of boundary conditions modeling action of self-straining actuators (smart material inclusions embedded in a wing).

The initial boundary value problem is reduced to a single evolution-convolution operator equation in the Hilbert state space of the problem. The spectral analysis of this equation is reduced to investigation of the distribution of the poles and structure of the residues at these poles for the so-called generalized resolvent operator. The generalized resolvent is an operator-valued finite meromorphic function of the spectral parameter. Its poles are exactly the aeroelastic modes and the residues at these poles describe the mode shapes.

The following results will be presented.

(i) Asymptotic representations of the eigenvalues and the eigenfunctions of the structural part of the model, i.e., asymptotic distribution of the natural frequencies of a wing. Riesz basis property of the eigenfunctions (or the mode shapes corresponding to the natural frequencies).

(ii) Asymptotic representations of the aeroelastic modes and the mode shapes for the entire model. Riesz basis property of the mode shapes.

(iii) Existence of a finite number of unstable aeroelastic modes responsible for flutter development and possible flutter control mechanisms will be discussed.

(iv) Comparison of analytical and computational results on the model.

Invertibility of half-plane Toeplitz operators with generating functions having homogeneous discontinuities

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This talk is aimed at Toeplitz operators $T_{+\cdot}(a) : l^2(\mathbb{Z}_+ \times \mathbb{Z}) \to l^2(\mathbb{Z}_+ \times \mathbb{Z})$ with generating functions having homogeneous discontinuities. Recall that a function $a \in L^\infty(\mathbb{T}^2)$ defined on the torus $\mathbb{T}^2$ has a homogenous discontinuity at the point $(s_0, t_0) \in \mathbb{T}^2$ if there exists a continuous function $\hat{a}_{(s_0,t_0)} \in C(\mathbb{T})$ such that

$$\lim_{p\to 0+0} \sup_{\varphi\in[0,2\pi)} \|a(s_0 e^{ip\cos\varphi}, t_0 e^{ip\sin\varphi}) - \hat{a}_{(s_0,t_0)}(e^{i\varphi})\|_{L^\infty(\mathbb{T}^2)} = 0\,.$$

Denote by $\gamma \subset L^\infty(\mathbb{T}^2)$ the class of all functions having homogenous discontinuities. Associate to $T_{+\cdot}(a)$, $a \in \gamma$, the family of Toeplitz operators $T(a_t) : l^2(\mathbb{Z}_+) \to l^2(\mathbb{Z}_+)$, $t \in \mathbb{T}$, where $a_t(s) = a(t,s)$ for $s \in \mathbb{T}$. For instance the following theorem holds:

Theorem. The operator $T_{+\cdot}(a), a \in \gamma$, is invertible if and only if

(1) for all $t \in \mathbb{T}$ the operator $T(a_t)$ is invertible,
(2) for all $t \in \mathbb{T}$ and $\tau \in \mathbb{T}$ the operators $T(a^+_{(\tau,t)}), T(a^-_{(\tau,t)}) : l^2(\mathbb{Z}_+) \to l^2(\mathbb{Z}_+)$ are invertible, where

$$a^\pm_{(\tau,t)}(e^{i\varphi}) = \hat{a}_{(\tau,t)}(e^{\pm i(\pi-\varphi)/2}) \text{ for } \varphi \in (-\pi,\pi)\,.$$

This result can be extended to matrix-valued generating functions γ_N. Moreover, the left or right invertibility of such operators can be completely described.

For continuous generating functions the results are well-known and go back to L. S. Goldenstein and I. Z. Gohberg (1960). The talk will also contain some retrospective on classical results of I. Gohberg and N. Krupnik concerning familiar Toeplitz operators with piecewise continuous generating functions.

Finally, let me mention that Toeplitz operators on half spaces with generating functions having homogenous discontinuities occur in the study of spline approximation methods for classes of two-dimensional singular integral operators.

Composition operators on function spaces related to Brennan's conjecture

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Brennan's conjecture concerns integrability of the derivative of a conformal map τ of the unit disk $\mathbb{D}$. The conjecture is that, for all such τ,

$$\int_{\mathbb{D}} (1/|\tau'|)^p\, dA < \infty$$

holds for $-2/3 < p < 2$. This was recently shown to be equivalent to compactness of certain weighted composition operators on the Bergman space $L^2_a(\mathbb{D})$. Here we introduce a class of analytic function spaces $L^2_a(\mu_p)$ on the simply connected domain $G = \tau(\mathbb{D})$ and prove that $L^2_a(\mu_p)$ supports compact (unweighted) composition operators if and only if $(\tau')^{-p} \in L^2_a(\mathbb{D})$. Hence Brennan's conjecture is equivalent to the existence of compact composition operators on $L^2_a(\mu_p)$ for p in the range $-1/3 < p < 1$. Motivated by this result, we study the boundedness and compactness of composition operators on the spaces we introduced. This is joint work with Valentin Matache.

Boundary relations and functional models

Henk de Snoo

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Let S be a closed symmetric relation (multivalued operator) in a Hilbert space $\mathfrak{H}$ and let $\mathcal{H}$ be an auxiliary Hilbert space. A linear relation Γ from $\mathfrak{H}^2 = \mathfrak{H} \times \mathfrak{H}$ to $\mathcal{H}^2 = \mathcal{H} \times \mathcal{H}$ is said to be a *boundary relation* for S^* if:

(1) $\mathcal{T} = \operatorname{dom} \Gamma$ is dense in S^* and the identity

$$(f', g)_{\mathfrak{H}} - (f, g')_{\mathfrak{H}} = (h', k)_{\mathcal{H}} - (h, k')_{\mathcal{H}},$$

holds for every $\{\widehat{f}, \widehat{h}\}$, $\{\widehat{g}, \widehat{k}\} \in \Gamma$;

(2) if $\{\widehat{g}, \widehat{k}\} \in \mathfrak{H}^2 \times \mathcal{H}^2$ satisfies the identity in (1) for every $\{\widehat{f}, \widehat{h}\} \in \Gamma$, then $\{\widehat{g}, \widehat{k}\} \in \Gamma$.

Here $\widehat{f} = \{f, f'\}$, $\widehat{g} = \{g, g'\} \in \mathfrak{H}^2$ and $\widehat{h} = \{h, h'\}$, $\widehat{k} = \{k, k'\} \in \mathcal{H}^2$. The notations $\mathfrak{N}_\lambda(\mathcal{T}) = \ker(\mathcal{T} - \lambda)$ and $\widehat{\mathfrak{N}}_\lambda(\mathcal{T}) = \{\, \widehat{f}_\lambda = \{f_\lambda, \lambda f_\lambda\} : f_\lambda \in \mathfrak{N}(\mathcal{T}) \,\}$ stand for the eigenspaces of the relation $\mathcal{T}$. The *Weyl family* $M(\lambda)$ of S corresponding to the boundary relation $\Gamma : \mathfrak{H}^2 \to \mathcal{H}^2$ is defined by $M(\lambda) = \Gamma\big(\widehat{\mathfrak{N}}_\lambda(\mathcal{T})\big)$, i.e.,

$$M(\lambda) = \big\{ \widehat{h} \in \mathcal{H}^2 \,:\, \{\widehat{f}_\lambda, \widehat{h}\} \in \Gamma \text{ for some } \widehat{f}_\lambda = \{f_\lambda, \lambda f_\lambda\} \in \mathfrak{H}^2 \big\}, \quad \lambda \in \mathbb{C} \setminus \mathbb{R}.$$

If the values $M(\lambda)$, $\lambda \in \mathbb{C} \setminus \mathbb{R}$, are operators, then one speaks of a *Weyl function*. Any Weyl family is a Nevanlinna family. Recall that a family $M(\lambda)$, $\lambda \in \mathbb{C} \setminus \mathbb{R}$, of linear relations in $\mathcal{H}$ is said to be a *Nevanlinna family* in $\mathcal{H}$ if

(a) $M(\lambda)$ is maximal dissipative (maximal accumulative) for $\lambda \in \mathbb{C}_+$ ($\lambda \in \mathbb{C}_-$);

(b) $M(\bar{\lambda}) = M(\lambda)^*$ for $\lambda \in \mathbb{C} \setminus \mathbb{R}$;

(c) for some, and hence for all, $\nu \in \mathbb{C}_+$ ($\nu \in \mathbb{C}_-$) the $\mathbf{B}(\mathcal{H})$-valued function $\lambda \mapsto (M(\lambda) + \nu)^{-1}$ is holomorphic on $\mathbb{C}_+$ ($\mathbb{C}_-$, respectively).

Each Nevanlinna family is a Weyl family of some boundary relation, which can be chosen *minimal*, i.e., $\mathfrak{H} = \operatorname{span}\{\, \mathfrak{N}(\mathcal{T}) : \lambda \in \mathbb{C}_+ \cup \mathbb{C}_- \,\}$. This realization result was proved by means of the Naimark dilation theorem (Derkach, Hassi, Malamud, and de Snoo); recently a corresponding functional model in terms of reproducing kernel Hilbert spaces was provided by Derkach, and, by different means, by Behrndt, Hassi, and de Snoo. In this talk the above notions will be discussed and it will be shown how the case of symmetric relations in indefinite inner product spaces can be treated.

Solution of affine parametric quadratic inverse eigenvalue problem via alternating projections method

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A new approach for an affine parametric inverse eigenvalue problem is proposed. Hybrid method combining Newton's method with quasi alternating projections method is applied, global convergence analysis is given. The new approach is capable of handling complex and real eigenvalues as well as bifurcation of eigenvalues. It is shown that locally method is quadratically convergent. Some numerical examples are presented. To apply quasi alternating projections method a matrix nearness problem is needed to be solved. Newton's method on the Stiefel manifold were developed to solve matrix nearness problem.

A direct approach to convolution type operators with symmetry

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We consider convolution type operators that carry a certain symmetry in their structure. The study is motivated by several applications in mathematical physics where this kind of operators appears. They can be regarded as a class of Wiener-Hopf plus Hankel operators acting in spaces of Bessel potentials. But the common approach of reduction to systems of Wiener-Hopf equations is avoided by a more direct factorization scheme. The main results are: Fredholm criteria, analytical representation of generalized inverses, and the constructive solution of normalization problems. For clarity we focus on the scalar case and summarize results on the matrix version.

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The Feichtinger Conjecture for exponential frames

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We consider the following special case of the Feichtinger Conjecture: Given a measurable set $E \subset [0,1]$ of positive measure, is there a finite partition $\{A_1, \dots, A_n\}$ of $\mathbb{Z}$ such that for each $1 \le j \le n$, $\{e^{2\pi i k x} 1_E : k \in A_j\}$ is a Riesz sequence? We will recall how this problem is related to the paving problem of Laurent operators, and report on some progress on this and related problems.

Hilbert spaces contained in quotients of Kreĭn spaces with applications to passive state/signal realization theory

Olof J. Staffans (presenter) and Damir Z. Arov

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Let $\mathcal{Z}$ be a maximal nonnegative subspace of a Kreĭn space $\mathfrak{K}$ with (indefinite) inner product $[\cdot,\cdot]_{\mathfrak{K}}$, let $\mathcal{Z}^{[\perp]}$ be the orthogonal companion to $\mathcal{Z}$ in $\mathfrak{K}$, and let $\mathcal{Z}_0 = \mathcal{Z} \cap \mathcal{Z}^{[\perp]}$ be the maximal neutral subspace of $\mathcal{Z}$. Then $[\cdot,\cdot]_{\mathfrak{K}}$ induces a positive inner product in the quotient space $\mathcal{Z}/\mathcal{Z}_0$, and $-[\cdot,\cdot]_{\mathfrak{K}}$ induces a positive inner product in the quotient space $\mathcal{Z}^{[\perp]}/\mathcal{Z}_0$. These two inner product spaces are not, in general, complete. We show that the completions of $\mathcal{Z}/\mathcal{Z}_0$ and $\mathcal{Z}^{[\perp]}/\mathcal{Z}_0$ can be identified in a natural way with certain subspaces of the quotient spaces $\mathfrak{K}/\mathcal{Z}^{[\perp]}$ and $\mathfrak{K}/\mathcal{Z}$, respectively. The construction of these subspaces is similar to the deBrange–Rovnyak construction used to realize an operator-valued Schur function in the unit disk $\mathbb{D}$ as the characteristic function of a discrete time input/state/output system. More precisely, the completion of $\mathcal{Z}^{[\perp]}/\mathcal{Z}_0$ can be identified with the following subspace $\mathcal{X}[\mathcal{Z}]$ of $\mathfrak{K}/\mathcal{Z}$. For each $k \in \mathfrak{K}$ we denote the equivalence class in $\mathfrak{K}/\mathcal{Z}$ to which k belongs by $[k] = k + \mathcal{Z}$. Then

$$\mathcal{X}[\mathcal{Z}] = \big\{[k] \in \mathfrak{K}/\mathcal{Z} \;\big|\; \|[k]\|_{\mathcal{X}[\mathcal{Z}]} < \infty\big\}, \tag{4}$$

where the norm $\|\cdot\|_{\mathcal{X}[\mathcal{Z}]}$ in $\mathcal{X}[\mathcal{Z}]$ is given by

$$\big\|[k]\big\|_{\mathcal{X}[\mathcal{Z}]} = \sqrt{\sup_{z \in \mathcal{Z}} (-[k-z, k-z]_{\mathfrak{K}})}. \tag{5}$$

The subspace $\mathcal{X}[\mathcal{Z}^{[\perp]}]$ of $\mathfrak{K}/\mathcal{Z}^{[\perp]}$ is defined in an analogous fashion.

We apply the technique described above to construct three canonical passive state/signal realizations of a given passive behavior $\mathfrak{W}$, namely a) a controllable forward conservative, b) an observable backward conservative, and c) a simple conservative state/signal realization. All of these are determined uniquely by $\mathfrak{W}$ up to unitary similarity. The passive behavior $\mathfrak{W}$ is roughly the time-domain counterpart of a shift-invariant maximal nonnegative subspace $\mathcal{Z}$ of the Kreĭn space $\mathfrak{K} := H^2(\mathbb{D}; \mathcal{W})$, where $\mathcal{W}$ is a Kreĭn space. By decomposing $\mathcal{W}$ in different ways into the direct sum of an input space and an output space and interpreting $\mathcal{Z}$ as the graph of a shift-invariant operator we get the standard input/state/output realizations of Schur functions, Carathéodory functions, and Potapov functions in the unit disk.

Reduction of a unitary matrix to quasiseparable form

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This work describes the computation of a similarity that reduces a unitary matrix to a quasiseparable form, represented as a product of rotations or Householder transformations. The algorithm is based on matrix vector products and is a generalization of the isometric Arnoldi algorithm for the reduction of a unitary matrix to unitary Hessenberg form. Given a unitary matrix A and a vector $\mathbf{x}$, the isometric Arnoldi algorithm computes an orthonormal sequence $\mathbf{q}_j$ such that if

$$Q = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \cdots & \mathbf{q}_n \end{bmatrix}$$

then $H = Q^* AQ$ is a unitary Hessenberg matrix which can be written as a product

$$H = G_0 G_1 G_2 \cdots G_n$$

where each G_k, $k < n$ is a modified plane rotation and $G_n = I \oplus \gamma_n$ with $|\gamma_n| = 1$. The generalized form of the algorithm computes an orthonormal sequence $\mathbf{q}_k$ such that

$$H = Q^* AQ = J_{n+1}^* J_n^* J_{n-1}^* \cdots J_0^* G_0 G_1 \cdots G_n$$

the J_k and G_k can be represented as products of plane rotations and/or Householder transformations. The rotations are computed in the course of applying a generalized isometric Arnoldi algorithm. The matrix H is no longer Hessenberg, but does have a quasiseparable structure.

Bubbles tend to the boundary

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Consider a Schrodinger operator on a bounded domain with an obstacle given by a compactly supported potential and impose Neumann boundary conditions. What is the optimal placement of the obstacle to minimize the ground state energy? We present joint work with J. Baker and M. Loss which shows for different classes of domains that the obstacle tends to the boundary. This is independent of the sign of the obstacle (i.e. attractive or repulsive potential), which is in contrast to the situation for the corresponding Dirichlet problem. We will also discuss applications of these results to the random displacement model, a random Schrodinger operator which requires optimal placement of infinitely many obstacles.

Complexifying the Krein space

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I persist in my efforts to campaign for extending the notion of Krein space by leaving apart selfadjointness of its fundamental symmetry. This was the subject of

> F. H. Szafraniec, "A look at Krein space: new thoughts and old truths," talk given at the Fifth Workshop on Operator Theory in Krein Spaces and Differential Equations, Technische Universität, Berlin, December 16–18, 2005,

followed by

> F. H. Szafraniec, "Two-sided weighted shifts are 'almost Krein' normal," *Operator Theory: Advances and Applications*, to appear.

I intend to discuss the topic once more, enriching it with drops of yet fresher flavour.

Optimization of the spectral radius of nonnegative matrices

Raymond Nung-Sing Sze
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Given a nonnegative matrix A. We show that there are permutation matrices P_1 and P_2 such that $\rho(P_1A) \leq \rho(SA) \leq \rho(P_2A)$ for all doubly stochastic matrices S, where $\rho(X)$ is the spectral radius of X. An additive version of the above problem is also considered and it is a surprise that the same situation holds, that is, there are permutation matrices Q_1 and Q_2 such that $\rho(Q_1 + A) \leq \rho(S + A) \leq \rho(Q_2 + A)$ for all doubly stochastic matrices S. Similar results are obtained for row / column stochastic matrices. Generalization to normal matrices with fixed spectral radius is also given.

This talk is based on a joint work with J. Axtell, L. Han, D. Hershkowitz and M. Neumann.

Extensions of Heinz's inequality and related inequalities for semisimple Lie groups

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Some extensions in the context of semisimple Lie groups of Heinz-Kato's inequalities and related inequalities, for example, McIntosh's inequality and Furuta inequality are discussed.

Presentation of the kernel of a matrix characteristic operator by the kernels of two operators one of them is a scalar characteristic operator

Anna Tarasenko (presenter) and Aleksandr Karelin

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We denote the Cauchy singular integral operator along a contour Γ by

$$(S_\Gamma \varphi)(x) = \frac{1}{\pi i} \int\limits_\Gamma \frac{\varphi(\tau)}{\tau - x} d\tau$$

and the identity operator on Γ by $(I_\Gamma \varphi)(t) = \varphi(t)$. We consider the operator

$$D_{\mathbb{R}_+} = uI_{\mathbb{R}_+} + vS_{\mathbb{R}_+},\ D_{\mathbb{R}_+} \in [L_2^2(\mathbb{R}_+, x^{-\frac{1}{4}})],$$

where

$$u(t) = \frac{1}{2} \begin{bmatrix} c(x) + d(x) & a(x) - 1 \\ c(x) + d(x) & a(x) + 1 \end{bmatrix}, \quad v(t) = \frac{1}{2} \begin{bmatrix} a(x) + 1 & c(x) - d(x) \\ a(x) - 1 & c(x) - d(x) \end{bmatrix}.$$

The following decomposition

$$\ker D_{\mathbb{R}_+} = \ker \tilde{H} \bigcap F \ker C,$$

is found.

Here operator C is a scalar characteristic operator, $C \in [L_2(\mathbb{R}_+)]$, operator F is invertible operator, $F \in [L_2(\mathbb{R}_+), L_2^2(\mathbb{R}_+, x^{-\frac{1}{4}})]$. Operators $\tilde{H}$ and C are constructed by an arbitrary nontrivial element of $\ker D_{\mathbb{R}_+}$ or by an arbitrary nontrivial element of the kernel of the associated operator.

A new hybrid method for finding an eigenpairs of a symmetric quadratic eigenvalue problem in an interval

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The symmetric quadratic eigenvalue problem

$$(\lambda^2 M + \lambda C + K)u = 0,$$

where M, C, and K are given $n \times n$ matrices and (λ, u) is an eigenpair, arises in a wide variety of practical applications, including vibration, acoustic, and noise control analysis. In the most practical application, the problem is often of a very large dimension. Unfortunately because of the nonlinearity, the problem is extremely hard to solve numerically. The state-of-art computational techniques, such as the Jacobi-Davidson method, are capable of computing only a few extremal eigenvalues and eigenvectors if the initial vector is chosen properly. Fortunately, there are engineering applications that require only some of the eigenvalues lying within an interval. In this paper, a new Hybrid method combining a Modified Parametrized Newton-type method with the Jacobi-Davidson method is proposed to compute an eigenpair of a quadratic pencil within an interval. The experimental results show that this method is much faster than the Jacobi-Davidson method. The results of this paper generalize those of an earlier work on Parameterized Newton's Algorithm for finding an eigenpair of a symmetric matrix.

Semiclassical limit of scattering transforms for the focusing NLS

Alexander Tovbis

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Direct and inverse scattering transforms are key ingredients in solving initial value problems for integrable evolution equations, such as KdV, NLS, SG and other famous equations. In the case of NLS, the scattering transform is defined through the corresponding Zakharov - Shabat (ZS) system of linear ODEs. In the semiclassical limit, ZS system is a singularly perturbed system of linear ODEs. We present expressions for leading order terms of direct and inverse scattering transforms that do not rely directly on WKB type of methods.

An F. and M. Riesz theorem for reproducing kernel Hilbert spaces

Tavan T. Trent

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When can a positive invertible operator, P, be factored as $P = A^*A$, where A is multiplication by an element of a reproducing kernel Hilbert space and A is invertible? We consider this question for reproducing kernel Hilbert spaces, including Hardy space on the polydisk in $\mathbb{C}^n$.

Block operator matrices with unbounded entries

Christiane Tretter

Institute of Mathematics, University of Bern, Switzerland

Operator matrices with unbounded entries, e.g. differential operators, are of great interest in applications like mathematical physics. In this talk different classes of such block operator matrices will be considered and methods to investigate their spectral properties will be presented.

Spectra of operators: theory, computations and applications

Christiane Tretter

Institute of Mathematics, University of Bern, Switzerland

In this talk methods for studying the spectral properties of block operator matrices will be presented. Particular emphasis is placed on block operator matrices to which standard spectral theory results do not readily apply, e.g. non-self-adjoint or non-semibounded block operator matrices. The questions addressed include localization of the spectrum and the essential spectrum, block diagonalizability, existence of solutions of algebraic Riccati equations, and corresponding Cauchy problems. Examples from mathematical physics will illustrate the results.

Generalized singular values

Jochen Trumpf (presenter) and Ram Somaraju

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We present a generalization of the usual Hilbert space concept of singular values for compact operators to the setting of arbitrary normed spaces.

The main idea behind the proposed generalization is to use finite dimensional exhaustion of the image W of the closed unit ball under the operator. For every given tolerance level $\varepsilon > 0$ there is a minimal (finite) dimensional subspace of the image space approximating W with accuracy ε in norm. This minimal dimension is known as the "number of degrees of freedom at level ε" in the theory of operator models for communication channels. The number of degrees of freedom is a non-increasing function of ε with a finite number of discontinuities in every given bounded ε-interval. We define the generalized singular values of the given operator to be the points of discontinuity of that function. This notion coincides with the usual concept of singular values in the special case of a Hilbert space setting.

We establish a number of basic results on generalized singular values, including some approximation results that lead to practical algorithms for their (approximate) computation. The latter require the existence of complete Schauder bases in the source and range spaces. We discuss an application to the question of fundamental capacity bounds on spatial communication channels as they appear in modern wireless multi-antenna systems. We will explain why Hilbert space models are not sufficiently general in this context and argue that compact operators between not necessarily complete normed spaces is the "correct" model class to use.

Normal and hyponormal matrices in inner product spaces

Carsten Trunk

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Complex matrices that are structured with respect to a possibly degenerate indefinite inner product are studied. Based on the theory of linear relations, the notion of an adjoint is introduced.

This notion is then used to give a new definition for normal matrices which allows the generalization of extension results for positive invariant subspaces from the case of nondegenerate inner products to the case of degenerate inner products.

Moreover, we study the class of hyponormal and strongly hyponormal matrices in inner product spaces with a possibly degenerate inner product. Finally, we describe the relation to Moore-Penrose normal matrices and normal matrices.

The talk is based on joint works with Mark-Alexander Henn (Berlin, Germany) and Christian Mehl (Birmingham, United Kingdom).

On non-self-adjoint Jacobi matrices

Eduard Tsekanovskiĭ

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We develop direct and inverse spectral analysis for finite and semi-infinite non-self-adjoint Jacobi matrices with a rank-one imaginary part. It is shown that given a set of n not necessarily distinct non-real numbers in the open upper (lower) half-plane uniquely determines a $n \times n$ Jacobi matrix with a rank-one imaginary part having those numbers as its eigenvalues counting algebraic multiplicity. An algorithm for reconstruction for such finite Jacobi matrices is presented. A new model complementing the well known Livsic triangular model for bounded linear operators with a rank-one imaginary part is obtained. It turns out that the model operator is a non-self-adjoint Jacobi matrix. This follows from the fact that any bounded, prime, non-self-adjoint linear operator with a rank-one imaginary part acting on some finite-dimensional (resp., separable infinite-dimensional Hilbert space) is unitarily equivalent to a finite (resp., semi-infinite) non-self-adjoint Jacobi matrix. This result strengthens a classical result of Stone established for self-adjoint operators with simple spectrum. We establish the non-self-adjoint analogs of the Hochstadt and Gesztesy–Simon uniqueness theorems for finite Jacobi matrices with non-real eigenvalues as well as an extension and refinement of these theorems for finite non-self-adjoint tri-diagonal matrices to the case of mixed eigenvalues, real and non-real. A unique Jacobi matrix, unitarily equivalent to the operator of integration $(Jf)(x) = 2\,i\,\int_x^l f(t)dt$ in the Hilbert space $L_2[0,l]$ is found as well as spectral properties of its perturbations and connections with the well known Bernoulli numbers. We also give the analytic characterization of the Weyl functions of dissipative Jacobi matrices with a rank-one imaginary part. The talk is based on joint work with Yu. Arlinskii.

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Structured matrices, continued fractions, and the generalized Routh-Hurwitz problem: Part II

Mikhail Tyaglov

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This is joint work with Olga Holtz.

We discuss application of the general theory to be presented by Olga Holtz to root localization of polynomials. In terms of Hurwitz and Hankel infinite matrices and Stieltjes continued fractions we describe a wide class of polynomials which includes classical Hurwitz polynomials and their dual self-interlacing polynomials as a marginal subclasses.

Composition operators on spaces of vector-valued functions

Hans-Olav Tylli

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Let D be the unit disk in the complex plane and φ an analytic self-map of D. I will describe some recent work, motivated by e.g. [4], [1], on properties of the composition operators C_φ; $f \mapsto f \circ \varphi$, on certain Banach spaces consisting of vector-valued analytic functions $f : D \to X$, where X is an infinite dimensional complex Banach space.

Topics to be discussed will include: (1) Qualitative properties of C_φ, such as weak compactness. (2) Characterization of the boundedness of $C_\varphi : wH^p(X) \to H^p(X)$ in terms of the Hilbert-Schmidt norm of $C_\varphi : H^2 \to H^2$ for $2 \leq p < \infty$, [3]. Here $H^p(X)$ is the strong X-valued Hardy space, and $wH^p(X)$ the corresponding weak X-valued Hardy space for which $\|f\|_{wH^p(X)} = \sup_{\|x^*\| \leq 1} \|x^* \circ f\|_{H^p} < \infty$, where $x^* \in X^*$. (3) Qualitative properties of the general "weighted" operator composition map $f \mapsto \psi(\cdot)(f(\varphi(\cdot)))$ on vector-valued H^∞-spaces, where $\psi : D \to L(X, Y)$ is an analytic operator-valued map [2]. Here $L(X, Y)$ is the space of bounded linear operators $X \to Y$.

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Some generalizations of Lieb-Thirring inequalities

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We extend the well known estimates for the negative eigenvalues of the Schrödinger operator to a wide class of Hamiltonians, which include in particular, operators on lattices, operators on free groups, on space of nilpotent matrices, on Lobachevsky plane, and on quantum graphs. We also will show that the Lieb-Thirring inequalities for standard Schrödinger operators with random potentials can be improved with probability one.

The first part is a joint work with Prof. S. Molchanov, the second part is a joint work with Prof. O. Safronov

Anatomy of the C^*-algebra generated by Toeplitz operators with piece-wise continuous symbols

Nikolai Vasilevski

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We give a detailed study of the C^*-algebra generated by Toeplitz operators with piece-wise continuous symbols acting on the Bergman space on the unit disk. We describe explicitly each operator from this algebra and characterize Toeplitz operators which belong to the algebra putting a special emphasis to Toeplitz operators with unbounded symbols. We show that none of a finite sum of finite products of the initial generators is a compact perturbation of a Toeplitz operator. At the same time the uniform closure of the set of such sums of products contains a huge amount of Toeplitz operators with bounded and unbounded symbols drastically different from symbols of the initial generators.

Noncommutative functions and their difference-differential calculus

Victor Vinnikov

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We define a function of d (free) noncommuting variables as a function from a domain in $(\mathbb{C}^{n\times n})^d$ to $\mathbb{C}^{n\times n}$, for all matrix dimensions n, satisfying natural compatibility conditions for different values of n (it has to respect direct sums, and to commute with joint similarity). Our main motivation came from the work of Helton and his coworkers on matrix convexity and matrix positivity, but as it turned out variants of this notion were already considered by J. L. Taylor in his work on functional calculus for noncommuting operators back in early 1970s. The main examples are provided by polynomials and power series in noncommuting variables, and our main result is a kind of noncommutative Taylor series under very weak regularity assumptions (in fact, local boundedness more or less suffices) which shows that in many natural situations this is everything. E.g., given a noncommutative function whose entries are polynomials in matrix entries of the arguments of uniformly bounded degree (with respect to the matrix dimension), this function equals a noncommutative polynomial. To prove these things we construct a kind of noncommutative differential calculus (more precisely, this calculus combines differential calculus and the calculus of finite differences). This should have many applications — e.g. we can establish easily various foundational results on singularities of noncommutative rational functions in terms of their minimal realizations which were previously known only in special situations and required a great amount of ingenuity and labour to establish.

This is a joint work with D. Kalyuzhnyi-Verbovetskii.

Embedding of pairs of commuting contractions into a conservative overdetermined 2*D* system

Victor Vinnikov

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The classical Schaeffer matrix construction of the minimal unitary dilation of a contraction can be viewed as consisting of two stages. First we embed the contraction as the state space operator into a conservative input/state/output system (the Halmos dilation). Then we embed this conservative input/state/output system into a scattering system.

For a pair of commuting contractions one can pursue a similar line of attack using conservative overdetermined 2D input/state/output systems and two-evolution scattering systems. The second stage — embedding a conservative overdetermined input/state/output system into the two-evolution scattering system of its admissible trajectories, is quite well understood. The first stage — the analogue of the Halmos dilation — turns out to be surprisingly intricate. I will discuss the background and recent progress.

This is a joint work with Joe Ball.

Toeplitz operators on Bergman spaces A^p

Jani Virtanen

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We study some (spectral) properties of Toeplitz operators T_a acting on Bergman spaces A^p with $1 < p < \infty$. In particular, we give sufficient conditions for boundedness, compactness and Fredholmness of T_a in terms of the averaging function $\hat{a}$. One of the key concepts of our study is the mean oscillation in the Bergman metric of a locally integrable function. This is joint work with Jari Taskinen.

Eigenvalue problems for the p-Laplacian with coefficients of bounded variation

Hans Volkmer

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We use generalized notions of total variation to estimate eigenvalues of the p-Laplacian.

Operator-valued Herglotz functions and Carathéodory-Fejér interpolation

Dan Volok

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Let B be a Banach space and let B^* denote its conjugate dual (that is, the space of anti-linear continuous functionals). Furthermore, let $\Phi(z)$ be a function defined on the open unit disk $\mathbb{D}$, whose values are continuous operators from B to B^*. The function $\Phi(z)$ is said to be a Herglotz function if the kernel

$$K_\Phi(z,w) = \frac{\Phi(z) + \Phi(w)^*|_B}{1 - zw^*}$$

is positive on $\mathbb{D}$. In this case the function $\Phi(z)$ is analytic on $\mathbb{D}$. For this class of functions we consider the classical Carathéodory-Fejér interpolation problem: *given* $N+1$ *continuous linear operators from* B *to* B^*, *say* $M_0, \ldots, M_N$, *find a Herglotz function* $\Phi(z)$ *such that*

$$\Phi(z) - \sum_{n=0}^{N} z^n M_n = O(z^{N+1}), \quad z \to 0.$$

We give a necessary and sufficient condition for this problem to be solvable and show that it can be reduced to the Hilbert space case.

Some related topics, such as realizations of Herglotz functions, are also considered. This is a joint work with D. Alpay and O. Timoshenko.

Oscillation theory for a quadratic eigenvalue problem

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We study the Sturm-Liouville problem with quadratic dependence on the spectral parameter,

$$-(p(x)y'(x))' + q(x)y(x) - \lambda s(x)y(x) = \lambda^2 y(x), \quad 0 \le x \le 1,$$

subject to boundary conditions $y'(0)\sin\alpha = y(0)\cos\alpha,\ \alpha \in [0,\pi)$, and $y'(1)\sin\beta = y(1)\cos\beta,\ \beta \in (0,\pi]$. Here, p, q, s are real valued functions satisfying $p > 0,\ q, 1/p \in L^1(0,1)$ and $s \in L^\infty(0,1)$. The eigenvalues are not necessarily real, but, as we show, all but a finite number of the eigenvalues are real and algebraically simple.

We are mainly concerned with determining the number of zeros in $(0,1)$ for the eigenfunctions associated with real eigenvalues using two parameter eigencurve theory. En route we develop asymptotic estimates for the eigenvalues.

Some algebraic properties of the Feichtinger conjecture for exponential frames

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The problem of paving Laurent operators and the related problem of the Feichtinger conjecture for exponential frames are difficult in part because of a lack of algebraic structure. We will place on these problems some algebraic structure via convolution, and discuss possible consequences of this. In particular, we can cast the problem of paving Laurent operators in terms of abstract Segal algebras, which may lead to advances in solving this problem.

3-Paving Small Matrices, experimental data for possible research directions

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The “paving parameters’” asymptotic behavior is equivalent to the Paving Problem and the Kadison-Singer Extension Problem. The single projection parameters, which measure the analogous bounds for single projections, are lower bounds for paving parameters. We have data for matrices up to size 25. For small sizes we have precise answers and for the larger, we have lower bounds which we find suggestive. This talk will explain the tables of values and what they may suggest about possible paving type conjectures and Bourgain’s single compression results. It is hoped that new information on paving small matrices might contribute to the larger problem.

The Hilbert-Schmidt norm and the commutator

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Suppose the space of $n \times n$-matrices is endowed with a norm $\|\cdot\|$. The question of interest is how small a constant C can be chosen such that

$$\|XY - YX\| \leq C\|X\|\|Y\|$$

is valid for all matrices X, Y. We will show that in this connection the Hilbert-Schmidt (or Frobenius) norm plays a distinguished role among all norms, we present several related inequalities, and we discuss the case of equality in the inequality above. The talk is based on joint work with A. Böttcher.

On the inverse and determinant of certain truncated Wiener-Hopf operators

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We find sharp asymptotics for the inverses and determinants of truncated Wiener-Hopf operators when the symbol has either one double zero or two simple zeros. We find formulas for the inverse that hold uniformly throughout the underlying interval with very small error, and formulas for the determinant with very small error.

Spectral problems for generalized indefinite strings

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Recall that an inverse spectral theorem of L. de Branges implies that each Nevanlinna function is the Titchmarsh-Weyl coefficient of a uniquely determined canonical system with some non-negative Hamiltonian matrix function H, and, according to M.G. Krein, each Stieltjes function is the principal Weyl coefficient of a uniquely determined string. Based on these results, it can be shown that each function Q from the class N_κ^+ (i.e. Q is a Nevanlinna function with κ negative squares such that the function $zQ(z)$ is a Nevanlinna function) is the Titchmarsh-Weyl coefficient of a uniquely determined generalized string where negative jumps and singularities in the mass distribution function and so-called dipols may appear.

A further generalization of indefinite strings with symmetric Nevanlinna functions with κ negative squares as Weyl coefficients is obtained by associated chains of symmetric Pontryagin spaces of entire functions being related to a chain of matrix functions playing the role of a generalized fundamental matrix. In particular, certain transformations of matrix functions are applied to obtain results about the structure of the singularities of generalized strings.

This is joint work with M. Kaltenbäck and H. Woracek.

Classes of tuples of commuting contractions satisfying the multivariable von Neumann inequality

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We obtain a decomposition for multivariable Schur-class functions on the unit polydisk which, to a certain extent, is analogous to Agler's decomposition for functions from the Schur–Agler class. As a consequence, we show that d-tuples of commuting strict contractions obeying an additional positivity constraint satisfy the d-variable von Neumann inequality for an arbitrary operator-valued bounded analytic function on the polydisk. Also, this decomposition yields a necessary condition for solvability of the finite data Nevanlinna–Pick interpolation problem in the Schur class on the unit polydisk.

This talk is based on joint work with Anatolii Grinshpan, Dmitry S. Kaliuzhnyi-Verbovetskyi and Victor Vinnikov.

Estimates of inverses of multivariable Toeplitz matrices

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The Gohberg-Semencul formula provides a formula for the inverse of a Toeplitz matrix based on the entries in the first and last columns of the inverse, under certain nonsingularity conditions. In this paper we study similar formulas for multivariable Toeplitz matrices, and we show that in the positive definite case these expressions provide upper bounds for the inverse in the Loewner order. Numerical experiments regarding the proximity of the estimate are included.

In addition, in a joint paper with graduate students Lei Cao and Selcuk Koyuncu, we use the above multivariable Gohberg-Semencul type expression as a starting point for an iterative algorithm to compute the inverse of the multivariable Toeplitz matrix. We will present some results of the numerical experiments.

Part of this talk is based on joint work with Lei Cao and Selcuk Koyuncu.

Domination in Krein spaces

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We provide two criteria for selfadjointness in a Krein space, which are closely connected with the notion of domination of unbounded operators. Recall that an operator A is said to *dominate B on the space* $\mathcal{E} \subseteq \mathcal{D}(A) \cap \mathcal{D}(B)$ if

$$\|Bf\| \leq c(\|f\| + \|Af\|), \qquad f \in \mathcal{E},$$

for some constant $c \geq 0$. The second important concept in the talk is the space of bounded vectors of an operator, defined as the union (taken over all $a > 0$) of the sets

$$\mathcal{B}_a(A) := \{f \in \mathcal{D}^\infty(A) : \exists c > 0 \quad \forall n \in \mathbb{N} \quad \|A^n f\| \leq ca^n\}.$$

First of the results ([2]) says the following. *If A is a symmetric operator in a Krein space $\mathcal{K}$ and there exists a sequence $(T_n)_{n=0}^\infty \subseteq \mathbf{B}(\mathcal{K})$ such that*

- $T_n \to I$ *in WOT,*
- $T_n(\mathcal{K}) \cup T_n^+(\mathcal{K}) \subseteq \mathcal{D}(A)$ *for* $n \in \mathbb{N}$,
- $\sup_{n\in\mathbb{N}} \|AT_n - T_nA\| < \infty$,

then A is selfadjoint. As an example of the operators T_n $(n \in \mathbb{N})$ one can take the projections onto the spaces $\mathcal{B}_n(S)$ of some selfadjoint operator S.

In the second theorem ([3]) it is assumed that a system of pointwise commuting symmetric operators in a Pontriagin space is given. Moreover, one of them is selfadjoint and dominates all the others. It appears that in such a situation all these operators are selfadjoint and that they commute spectrally. The result is an indefinite inner product version of a theorem from [1]. In both papers the bounded vectors turned up as a very useful tool in the proof.

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M-functions for closed extensions of adjoint pairs of operators

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We generalise the Weyl m-function from Sturm-Liouville problems and the Dirichlet-to-Neumann map from PDEs to the setting of adjoint pairs of operators. We show that in this setting every closed extension of an operator is associated with an abstract M-function and discuss spectral properties of the operator via the M-function. The results can be applied to elliptic PDEs.

The index problem for Sturm-Liouville eigenvalues for coupled boundary conditions

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Many important operators are self-adjoint and have a spectrum that is discrete and bounded from below. Examples of such operators include the self-adjoint Sturm-Liouville operators (SLOs) with integrable coefficient functions and a positive leading coefficient function. For such operators, there is the so-called index problem: given an eigenvalue of such an operator, how to determine its index (if it is simple) or indices (if it is not simple)?

In addition to its obvious theoretical interest, the index problem also has strong practical and computational motivations. For SLOs with separated boundary conditions (BCs), the index problem has been solved a long time ago, using the well-known Prüfer angle method based on oscillation properties of eigenfunctions. However, even for SLOs with *coupled* BCs, the index problem has not been solved.

In this talk, we give a complete solution of the index problem for SLOs with coupled BCs. As an application, we show that Fulton's conjecture about the index of a concrete eigenvalue of a class of explicit SLOs is always correct. Moreover, using a set of Mathematica codes, we demonstrate that indices of eigenvalues of SLOs, with either separated or coupled BCs, can be computed in real time.

Parts of this work were completed jointly with Zhong Wang or with Guixia Wang and Zhong Wang.

Singular integral operators and essential commutativity on the sphere

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Let $\mathcal{T}$ be the C^*-algebra generated by the Toeplitz operators $\{T_\varphi : \varphi \in L^\infty(S, d\sigma)\}$ on the Hardy space $H^2(S)$ of the unit sphere in $\mathbf{C}^n$. It is well known that $\mathcal{T}$ is contained in the essential commutant of $\{T_\varphi : \varphi \in \mathrm{VMO}\cap L^\infty(S, d\sigma)\}$. We show that the essential commutant of $\{T_\varphi : \varphi \in \mathrm{VMO}\cap L^\infty(S, d\sigma)\}$ is strictly larger than $\mathcal{T}$. Since the case $n = 1$ has already appeared in publication [Trans. AMS **360** (2008), 1089-1102], this talk will focus on the case $n \geq 2$, which is a more recent result.

Operator inequalities obtained from Uchiyama's results on operator monotonicity

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We shall consider operator inequalities of the form $\varphi(g(B)^{\frac{1}{2}}h(A)g(B)^{\frac{1}{2}}) \geq h(B)g(B)$ and $\varphi(g(B)^{\frac{1}{2}}h(A)g(B)^{\frac{1}{2}}) \geq g(B)^{\frac{1}{2}}\hat{h}(A)g(B)^{\frac{1}{2}}$ including Furuta inequality by applying M. Uchiyama's recent results on operator monotone functions.

Projective spectrum in Banach algebras

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For a tuple $A = (A_1, \ ..., \ A_n)$ of elements in a unital Banach algebra $\mathcal{B}$, its *projective spectrum* $p(A)$ is defined to be the collection of $z = [z_1, \ ..., \ z_n] \in \mathbf{P}^{n-1}$ such that $A(z) = z_1A_1 + z_2A_2 + \cdots + z_nA_n$ is not invertible in $\mathcal{B}$. The pre-image of $p(A)$ in C^n is denoted by $P(A)$. We will see that when $\mathcal{B}$ is a C^*-algebra, the *projective resolvent set* $P^c(A) := \mathbb{C}^{n+1} \setminus P(A)$ consists of domains of holomorphy. Maurer-Cartan type $\mathcal{B}$-valued holomorphic 1-form $A^{-1}(z)dA(z)$ on $P^c(A)$ contains some topological information about $P^c(A)$. For instance, if $\mathcal{B}$ is a C^*-algebra with a trace ϕ, then $\phi(A^{-1}(z)dA(z))$ is a nontrivial element in the de Rham cohomology space $H^1_d(P^c(A), \ C)$. Many examples will be given in this talk.

Subspaces with a common complement in a Banach space

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We study the problem of the existence of a common algebraic complement for a pair of closed subspaces of a Banach space. We prove the following two characterizations: (1) The pairs of subspaces of a Banach space with a common complement coincide with those pairs which are isomorphic to a pair of graphs of bounded linear operators between two other Banach spaces. (2) The pairs of subspaces of a Banach space X with a common complement coincide with those pairs for which there exists an involution S on X exchanging the two subspaces, such that $I + S$ is bounded from below on their union. Moreover we show that, in a separable Hilbert space, the only pairs of subspaces with a common complement are those which are either equivalently positioned or not completely asymptotic to one another. We also obtain characterizations for the existence of a common complement for subspaces with closed sum.

This is a joint work with D. Drivaliaris

Which 2-hyponormal 2-variable weighted shifts are subnormal?

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It is well known that a 2-hyponormal unilateral weighted shift with two equal weights must be flat, and therefore subnormal. By contrast, a 2-hyponormal 2-variable weighted shift which is both horizontally flat and vertically flat need not be subnormal. In this paper we identify a large class $\mathcal{S}$ of flat 2-variable weighted shifts for which 2-hyponormality is equivalent to subnormality. One measure of the size of $\mathcal{S}$ is given by the fact that within $\mathcal{S}$ there are hyponormal shifts which are not subnormal.

This is joint work with Raul E. Curto and Sang Hoon Lee.

Standard pairs of matrix polynomials and vibrating systems

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Matrix polynomials and linear time-invariant control systems of the form

$$\dot{x}(t) = Ax(t) + Bu(t)$$

are closely related. The link is the concept of standard pair. Standard pairs were introduced in [3] (see also [4] for a definition with a more local flavor). However there are several equivalent ways of defining them including one related to the strict system equivalence as introduced by Rosenbrock ([5]) or Fuhrmann ([1, 2]). Advantage of these characterizations can be taken to show that there is a one-to-one correspondence between classes of similarity systems and classes of right equivalent non-singular matrix polynomials. A similar result about feedback equivalence of systems and Wiener–Hopf equivalence of matrix polynomials can be exhibited.

On the other hand, a quadratic matrix polynomial $L(s) = Ms^2 + Ds + K$, with $\det M \neq 0$, can be seen as a model for a vibrating system. Two such systems are isospectral if they have the same Smith normal form. Any two isospectral systems are related by *filters*. That is to say, if $L_1(s)$ and $L_2(s)$ are isospectral systems then there are matrices $F_1(s) = F_{11}s + F_{12}$ and $L_2(s) = F_{21}s + F_{22}$ such that

$$L_1(s)F_2(s) = F_1(s)L_2(s)$$

Filters play a major role in the problem of decoupling a system (when possible). It will be shown that standard pairs can be used to parameterize all filters between two given systems.

Part of this work has been done in collaboration with A. Amparan and S. Marcaida.

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3-paving small matrices and the Kadison-Singer extension problem

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The Kadison-Singer Problem (KS), open since 1959, asks whether every pure state on ℓ^∞ extends uniquely to a pure state on $B(\ell^2)$. Owing to work of Anderson, it is known that KS is equivalent to the Matrix Paving Problem:

> Does there exists an $\varepsilon > 0$ and an integer k such that every zero-diagonal $n \times n$ matrix "k-paves to $1 - \varepsilon$"?

In spite of significant progress by Berman-Halpern-Kaftal-Weiss and Bourgain-Tzafriri, the Matrix Paving Problem (and KS) remains unsolved. In this talk, based on joint work with Gary Weiss of the University of Cincinnati, we examine the Matrix

Paving Problem for small parameter values: $k = 3$ and $n \leq 16$. We show that every 4×4 (resp. 5×5) zero-diagonal matrix 3-paves to $\frac{2}{1+\sqrt{5}} \approx 0.6180$, that every 6×6 zero-diagonal matrix 3-paves to $\frac{1}{\sqrt{2}} \approx 0.7071$, and that these results are sharp. On the other hand, we produce a 13×13 unitary circulant that 3-paves to approximately 0.8615. Our techniques combine operator theory, graph theory, and computer experimentation.

Composition operators and closures of some Mobius invariant spaces in the Bloch space

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We characterize bounded and compact composition operators from the Bloch space to the closure of some Mobius invariant subspaces, including BMOA and Q_p spaces, in the Bloch space. This is a joint work with Rauno Aulaskari.

Classifications of recurrence relations for polynomials via subclasses of Hessenberg-quasiseparable matrices of arbitrary order

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The results on characterization of orthogonal polynomials and Szegö polynomials via tridiagonal matrices and unitary Hessenberg matrices, resp., are classical. In a recent paper we observed that tridiagonal matrices and unitary Hessenberg matrices both belong to a wide class of $(H, 1)$-quasiseparable matrices and derived a complete characterization of the latter class via polynomials satisfying certain EGO-type recurrence relations. We also established a characterization of polynomials satisfying three-term recurrence relations via $(H, 1)$-well-free matrices and of polynomials satisfying the *Szegö*-type two-term recurrence relations via $(H, 1)$-semiseparable matrices. In this talk we present the generalization of all these results from scalar $(H, 1)$ to the block (H, m) case. Specifically, we provide a complete characterization of (H, m)-quasiseparable matrices via polynomials satisfying block EGO-type two-term recurrence relations. Further, (H, m)-semiseparable matrices are completely characterized by the polynomials obeying block *Szegö*-type recurrence relations. Finally, we completely characterize polynomials satisfying m-term recurrence relations via a new class of matrices called (H, m)-well-free matrices. This is joint with T.Bella and V.Olshevsky.

Isometric composition operators on Bloch-type spaces

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The isometric composition operators on the classical Bloch space have been described in the recent works of F. Colonna, M. Martin and D. Vukotic. We will show that the only isometric composition operators on the other Bloch-type spaces are induced by rotations. The same is also true for a number of cases when the composition operator acts between two different Bloch-type spaces.

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