Math 211 - Linear Algebra

True/False Solution Examples

In the True/False problems in the textbook, you need to give complete explanations, and not just the word "True" or "False". The following are examples of complete, correct solutions to a few of these problems.

Section 1.1, page 12, Problem 24.
a. Elementary row operations on an augmented matrix never change the solution set of the associated linear system.
Solution: True. When performing an elementary row operation to an augmented matrix, this is the same as algebraically manipulating the corresponding linear system to obtain a linear system which has the same solutions (this is stated on page 8).

b. Two matrices are row equivalent if they have the same number of rows.
Solution: False. The definition of two matrices being row equivalent is that elementary row operations may be performed on one to obtain the other. For example, if we take the two 1-by-3 matrices $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, then these have the same number of rows (just one row), but there are no elementary row operation that can be performed on one to obtain the other.

c. An inconsistent system has more than one solution.
Solution: False. The definition of an inconsistent system (as given on page 4) is that it has no solutions.

d. Two linear systems are equivalent if they have the same solution set.
Solution: True. The defintion of two linear systems being equivalent is precisely that they have the same solution set (as given on page 3).

Section 1.3, page 38, Problem 23.
a. Another notation for the vector $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$ is $\begin{bmatrix} -4 & 3 \end{bmatrix}$.

Solution: False. The first is a vector in $\mathbb{R}^2$, and so is a 2-by-1 matrix, while the second is a 1-by-2 matrix, and in particular is not a vector in $\mathbb{R}^2$.

b. The points in the plane corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ lie on a line through the origin.
Solution: False. The line through the origin which contains the point corresponding to $\mathbf{v} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ is exactly the set of points corresponding to Span$\{\mathbf{v}\}$ (as discussed on page 35). So, in order for the point corresponding to $\mathbf{w} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$ to be on the same line, we would need for $\mathbf{w}$ to be an element of Span$\{\mathbf{v}\}$, and so we would need $\mathbf{w}$ to be a scalar multiple of $\mathbf{v}$. But, if there was a scalar c such that $c\mathbf{v} = \mathbf{w}$, then we would have, by equating entries, $-2c = -5$ and $5c = 2$, which is impossible.

c. An example of a linear combination of vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is the vector $\frac{1}{2}\mathbf{v}_1$.
Solution: True. That is, as long as $\mathbf{v}_1$ and $\mathbf{v}_2$ are both in $\mathbb{R}^n$. The linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$ with scalars (or weights) c_1 and c_2 is the vector $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$, by the definition on page 32. So, taking $c_1 = \frac{1}{2}$ and $c_2 = 0$, we get that one linear combination is $\frac{1}{2}\mathbf{v}_1 + 0\mathbf{v}_2 = \frac{1}{2}\mathbf{v}_1$.

d. The solution set of the linear system whose augmented matrix is $[\mathbf{a}_1 \ \ \mathbf{a}_2 \ \ \mathbf{a}_3 \ \ \mathbf{b}]$ is the same as the solution set of the equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{b}$.
Solution: True. This follows exactly from what is stated on page 34.

e. The set Span$\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.
Solution: False. For example, if $\mathbf{v}$ is a scalar multiple of $\mathbf{u}$, then Span$\{\mathbf{u}, \mathbf{v}\} =$ Span$\{\mathbf{u}\}$, which is visualized as a line, and not a plane, when $\mathbf{u} \neq \mathbf{0}$.