Quiz 3 **Solutions**, Math 211, Section 1 (Vinroot)

(a): If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, compute $2A - 3B$.

Solution: $2A - 3B = 2\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - 3\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 6 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 6 & -2 \end{bmatrix}$.

(b): If B is a 3×7 matrix, and A is a 3×5 matrix, what is the size is $A^T B$? Explain briefly.

Solution: Since A is 3×5, then A^T is 5×3. Then $A^T B$ is a 5×3 matrix multiplied by a 3×7 matrix, which results in a 5×7 matrix. So $A^T B$ is a 5×7 matrix.

(c): If we were given B and A with sizes as in part **(b)**, could we compute BA? Why or why not?

Solution: We could not compute BA. Since B is a 3×7 matrix, and A is a 3×5 matrix, then BA is not defined.

(d): If $T : \mathbb{R}^2 \to \mathbb{R}^2$ and $S : \mathbb{R}^2 \to \mathbb{R}^2$ are linear transformations with standard matrices $C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, respectively, what is the standard matrix of the linear transformation $T \circ S : \mathbb{R}^2 \to \mathbb{R}^2$?

Solution: The standard matrix for $T \circ S$ is the product CD. In fact, this is how we motivated the definition of the product CD. We compute CD:

$$CD = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + (-1) \cdot 2 & 1 \cdot 0 + (-1) \cdot 1 \\ 0 \cdot 1 + 1 \cdot 2 & 0 \cdot 0 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}.$$