Quiz 0 **Solutions**, Math 211, Section 1 (Vinroot)

Consider the following linear system of equations, which geometrically represents three planes:

$$\begin{array}{rcrcrcr} -x_1 & - & 2x_2 & + & 2x_3 & = & 2 \\ x_1 & + & 2x_2 & & & = & -3 \\ -2x_1 & - & 4x_2 & + & 2x_3 & = & 1 \end{array}$$

Use an augmented matrix and the row reduction algorithm to determine whether these three planes have any point in common. Be sure to explain your conclusion.

Solution: First, the three planes have at least one point in common precisely when there is at least one solution to the system of linear equations. In other words, there is no point of intersection common to all three planes if and only if the linear system is inconsistent. The augmented matrix for the system is $\left[\begin{array}{rrr|r} -1 & -2 & 2 & 2 \\ 1 & 2 & 0 & -3 \\ -2 & -4 & 2 & 1 \end{array}\right]$ (the book would omit the vertical line separating the coefficients from the constants on the other side of the equations). We now use elementary row operations and our row reduction algorithm to put the matrix into row echelon form (we will not need to put it into *reduced* row echelon form though, as we will see).

We see that there is already a nonzero entry in the first entry of the first row, which is our first pivot position, so we use the first row to eliminate the nonzero entries below that pivot. We accomplish this by replacing R2 by R2+R1, and replacing R3 by R3−2(R1). The result after these row operations is:

$$\left[\begin{array}{rrr|r} -1 & -2 & 2 & 2 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -2 & -3 \end{array}\right].$$

In the second row, we see that neither the first nor second positions can be pivots, since they are 0's, and have 0's below them. So, the third entry in the second row is in our next pivot position. We eliminate the nonzero entry below it, using the second row, by replacing R3 by R3+R2. The resulting matrix is:

$$\left[\begin{array}{rrr|r} -1 & -2 & 2 & 2 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -4 \end{array}\right].$$

The augmented matrix is now in row echelon form (not *reduced*, though). The last row of this augmented matrix corresponds to "$0 = -4$", which indicates that the system must be inconsistent. Thus, this system of linear equations has no solutions, and so the three planes have no point in common.