Quiz 8 **Solutions**, Math 112, Section 1 (Vinroot)

Determine whether the following series converge or diverge. Explain and show steps clearly for full credit.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{n^4+3}$

Solution: Note that all of the terms of this series are positive, and so we may consider using the Comparison Test or the Limit Comparison Test. By looking at the highest powers of n, you should guess that this series should behave like $\sum_{n=1}^{\infty} \frac{1}{n^3}$, which converges since it is a p-series with $p = 3 > 1$. If we use the Comparison Test, we note $2n - 1 < 2n$, and since $n^4 + 3 > n^4$, then $\frac{1}{n^4+3} < \frac{1}{n^4}$. We thus have

$$\frac{2n-1}{n^4+3} < \frac{2n}{n^4} = \frac{2}{n^3},$$

and since $\sum_{n=1}^{\infty} \frac{2}{n^3} = 2\sum_{n=1}^{\infty} \frac{1}{n^3}$, then this series also **converges** by the Comparison Test.

If we use the Limit Comparison Test, we can compute

$$\lim_{n\to\infty} \frac{(2n-1)/(n^4+3)}{1/n^3} = \lim_{n\to\infty} \frac{n^3(2n-1)}{n^4+3} = \lim_{n\to\infty} \frac{2n^4-n^3}{n^4+3} \cdot \frac{1/n^4}{1/n^4} = \lim_{n\to\infty} \frac{2-1/n}{1+3/n^4} = 2 > 0.$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges, then this series also **converges** by the Limit Comparison Test.

2. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3}{4n-1}$

Solution: This series is an alternating series, and so we use the Alternating Series Test. We first find

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{3}{4n-1} = 0,$$

since the denominator gets arbitrarily large and the numerator is constant. We also have $4(n+1) - 1 = 4n + 3 > 4n - 1$, and so

$$b_{n+1} = \frac{3}{4(n+1)-1} = \frac{3}{4n+3} < \frac{3}{4n-1} = b_n.$$

The conditions of the Alternating Series Test hold, and so this series **converges**.