Quiz 5 **Solutions**, Math 112, Section 1 (Vinroot)

1. Find the length of the graph $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 1$. Show all of your steps clearly and carefully to receive full credit.

Solution: The formula for the length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

$$L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx.$$

In this problem, we have $f(x) = \frac{2}{3}x^{3/2}$, so $f'(x) = x^{1/2}$, and $1 + (f'(x))^2 = 1 + x$. The length of the graph from $x = 0$ to $x = 1$ is

$$L = \int_0^1 \sqrt{1 + x} \, dx.$$

Make the substitution $u = 1 + x$, so $du = dx$, and $u = 1 + 0 = 1$ when $x = 0$ and $u = 1 + 1 = 2$ when $x = 1$. Now the length of the graph is

$$L = \int_1^2 \sqrt{u} \, du = \frac{2}{3}u^{3/2}\Big]_1^2 = \frac{2}{3}(\sqrt{2^3} - \sqrt{1^1}) = \frac{4\sqrt{2}}{3} - \frac{2}{3}.$$