Quiz 2 **Solutions**, Math 112, Section 1 (Vinroot)

Compute each of the following. Show all of your steps clearly to receive full credit.

1. Find the average value of the function $f(x) = \cos(x)e^{\sin(x)}$ on the interval $[0, \pi/2]$.

Solution: The average value of $f(x)$ on $[0, \pi/2]$ is given by

$$\frac{1}{\pi/2 - 0} \int_0^{\pi/2} \cos(x)e^{\sin(x)} \, dx.$$

Make the substitution $w = \sin(x)$, so $dw = \cos(x) \, dx$. When $x = 0$, $w = \sin(0) = 0$, and when $x = \pi/2$, $w = \sin(\pi/2) = 1$. So the average value is now

$$\frac{2}{\pi} \int_0^1 e^w \, dw = \frac{2}{\pi}(e^w)\Big|_0^1 = \frac{2}{\pi}(e - 1).$$

2. Compute $\displaystyle\int x \cos(3x) \, dx$.

Solution: We use integration by parts, letting $u = x$ so $du = dx$, and $dv = \cos(3x) \, dx$ so $v = \frac{1}{3}\sin(3x)$. We then have

$$\int x \cos(3x) \, dx = \frac{1}{3}x \sin(3x) - \int \frac{1}{3}\sin(3x) \, dx = \frac{1}{3}x \sin(3x) + \frac{1}{9}\cos(3x) + C.$$