Quiz 4 **Solutions**, Math 112, Section 2 (Vinroot)

For each improper integral below, either compute its exact value, or show that it diverges. Show all of your steps clearly.

(a): $\displaystyle\int_{-\infty}^{\ln 2} e^{2x}\, dx.$

Solution: We compute using the definition of an improper integral on an infinite interval:

$$\begin{aligned}\int_{-\infty}^{\ln 2} e^{2x}\, dx &= \lim_{t\to-\infty}\int_{t}^{\ln 2} e^{2x}\, dx \\ &= \lim_{t\to-\infty}\left[\frac{1}{2}e^{2x}\right]_t^{\ln 2} \\ &= \lim_{t\to-\infty}\left(\frac{1}{2}e^{2\ln 2}-\frac{1}{2}e^{2t}\right) \\ &= \frac{1}{2}e^{\ln 4}-\frac{1}{2}\lim_{t\to-\infty} e^{2t} = 2-\frac{1}{2}(0) = 2.\end{aligned}$$

(b): $\displaystyle\int_{1}^{2} \frac{1}{(x-1)^{1/4}}\, dx.$

Solution: By the definition of an improper integral on an interval with a discontinuity, we have:

$$\begin{aligned}\int_{1}^{2} \frac{1}{(x-1)^{1/4}}\, dx &= \lim_{t\to1^+}\int_{t}^{2} \frac{1}{(x-1)^{1/4}}\, dx \\ &= \lim_{t\to1^+}\left[\frac{4}{3}(x-1)^{3/4}\right]_t^{2} \\ &= \lim_{t\to1^+}\left(\frac{4}{3}1^{3/4}-\frac{4}{3}(t-1)^{3/4}\right) \\ &= \frac{4}{3}-\frac{4}{3}\lim_{t\to1^+}(t-1)^{3/4} \\ &= \frac{4}{3}-\frac{4}{3}(1-1)^{3/4} = \frac{4}{3}.\end{aligned}$$