Quiz 3 **Solutions**, Math 112, Section 2 (Vinroot)

Compute the following indefinite integral. Show all steps in your computation in a clear and organized way.

$$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$$

Solution: Make the trigonometric substitution $x = 2\sin\theta$, so $dx = 2\cos\theta\,d\theta$. Then $x^2 = 4\sin^2\theta$, and $\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = \sqrt{4\cos^2\theta} = 2\cos\theta$. Now the integral transforms as follows:

$$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{2\cos\theta}{4\sin^2\theta\, 2\cos\theta}\,d\theta = \int \frac{1}{4}\csc^2\theta\,d\theta.$$

Using the fact that the antiderivative of $\csc^2\theta$ is $-\cot\theta$, we have

$$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{4}\csc^2\theta\,d\theta = -\frac{1}{4}\cot\theta + C.$$

Finally, we must write this expression in terms of x. Since $x = 2\sin\theta$, so $\sin\theta = \frac{x}{2}$, we can picture θ as an angle in a right triangle with hypoteneuse length 2 and side length opposite the angle being x. Then the other side must have length $\sqrt{4-x^2}$. Thus $\tan\theta = x/\sqrt{4-x^2}$, and $\cot\theta = \sqrt{4-x^2}/x$. So, the final form of the antiderivative is:

$$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = -\frac{1}{4}\cot\theta + C = -\frac{\sqrt{4-x^2}}{4x} + C.$$