Quiz 0 **Solutions**, Math 111, Section 1 (Vinroot)

(a): Compute the following limit if it exists, and if it does not exist but is infinite, describe the infinite limit and explain (factor the denominator):

$$\lim_{x \to -1^-} \frac{x-2}{x^2 - 3x - 4}$$

Solution: First note that the denominator is 0 if we try to substitute $x = -1$, and so we cannot directly substitute. Also, the numerator does not go to 0, and so the limit should be infinite. Factoring the denominator gives $x^2 - 3x - 4 = (x+1)(x-4)$. When $x \to -1^-$, then $x < -1$, which means $x + 1 < 0$ while approaching 0. For the other factors, $(x - 2) \to -3$ and $(x - 4) \to -5$ as $x \to -1^-$. Since all three factors are negative when x approaches -1 from the left, then the entire expression is negative. Since the denominator goes to 0 while the numerator does not, we have

$$\lim_{x \to -1^-} \frac{x-2}{x^2 - 3x - 4} = -\infty.$$

(b): Compute the following limit (if it exists), making your steps clear: $\lim_{x \to 0} \sqrt[5]{\frac{x^3 - 1}{3x^2 + 2x + 1}}$.

Solution: By properties of limits we have stated, we have

$$\lim_{x \to 0} \sqrt[5]{\frac{x^3 - 1}{3x^2 + 2x + 1}} = \sqrt[5]{\lim_{x \to 0} \frac{x^3 - 1}{3x^2 + 2x + 1}} = \sqrt[5]{\frac{0^3 - 1}{3(0^2) + 2(0) + 1}} = \sqrt[5]{-1} = -1.$$

Note that since we are taking the fifth root, and not an even root, then there is no danger of the fifth root not being a real number. Also, since substituting in $x = 0$ in the denominator does not yield 0, then the direct substitution in the rational expression is also valid.