Quiz 0 **Solutions**, Math 111, Section 4 (Vinroot)

(a): Compute the following limit (if it exists), making your steps clear: $\lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$.

Solution: We cannot plug in $h = 0$, because the expression is not defined there. So we must algebraically manipulate the function inside of the function by finding the common denominator of $3(3+h)$:

$$\lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h\to 0} \frac{\frac{3-(3+h)}{3(3+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{3(3+h)}}{h} = \lim_{h\to 0} \frac{-h}{3h(3+h)}.$$

Now note that as $h \to 0$, that in particular $h \neq 0$, and so we may cancel the factor of h in the numerator and denominator inside of the limit. We now have

$$\lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h\to 0} \frac{-1}{3(3+h)} = \frac{-1}{3(3+0)} = -\frac{1}{9}.$$

(b): Compute the following limit if it exists, and if it does not exist but is infinite, describe the infinite limit and explain (factor the denominator):

$$\lim_{x\to 1^-} \frac{x-4}{x^2 - 3x + 2}$$

Solution: First note that $x^2 - 3x + 2 = (x-2)(x-1)$, and so we cannot plug in $x = 1$ since the function is not defined there. Also, the numerator is *not* 0 when we let $x = 1$, so we should not expect a cancellation as in **(a)** above. Instead, since the denominator goes toward 0 and the numerator does not, we expect some infinite limit.

When $x \to 1^-$, the numerator approaches -3, and in particular is a negative number. In the denominator, as $x \to 1^-$ we have $x - 2$ approaches -1, and so is negative. As $x \to 1^-$, then $x < 1$, and so $x - 1$ is negative and approaches 0. Putting this together, as $x \to 1^-$, the denominator $(x-2)(x-1)$ is a negative times a negative number, and so positive, and approaches 0. That is, as $x \to 1^-$, $\frac{1}{x^2 - 3x + 2}$ becomes a large positive number. Meanwhile, the numerator approaches -3, and so the whole expression tends towards the product of -3 times a large positive number, which gives a very negative number. In other words, the limit does not exist, and we have

$$\lim_{x\to 1^-} \frac{x-4}{x^2 - 3x + 2} = -\infty.$$