Quiz 7 **Solutions**, Math 111, Section 4 (Vinroot)

For each of the following, say what indeterminate form appears (at any step when there is one), and evaluate. Show your steps clearly.

1. $\displaystyle\lim_{x\to 0} \frac{xe^x}{e^x - 1}$

Solution: We have $xe^x \to 0$ and $(e^x - 1) \to 0$ as $x \to 0$, and so this is an indeterminate form of type $\frac{0}{0}$. We apply L'Hospital's rule, and we have:

$$\lim_{x\to 0} \frac{xe^x}{e^x - 1} = \lim_{x\to 0} \frac{e^x + xe^x}{e^x} = \lim_{x\to 0} \frac{e^x(1+x)}{e^x} = \lim_{x\to 0}(1+x) = 1.$$

Note that after we applied L'Hospital's rule by taking the derivative of the numerator and denominator, there was a common factor of e^x in the numerator and denominator that we could cancel. So we have $\displaystyle\lim_{x\to 0} \frac{xe^x}{e^x - 1} = 1$.

2. $\displaystyle\lim_{x\to\infty} x\sin(1/x)$

Solution: As $x \to \infty$, we have $(1/x) \to 0$, and so $\sin(1/x) \to \sin(0) = 0$. That is, this is an indeterminate form of type $\infty \cdot 0$. We take $x = \frac{1}{1/x}$ to turn this into a quotient, that is

$$\lim_{x\to\infty} x\sin(1/x) = \lim_{x\to\infty} \frac{\sin(1/x)}{1/x},$$

which is an indeterminate form of type $\frac{0}{0}$ since $1/x \to 0$ as $x \to \infty$. We then apply L'Hospital's rule:

$$\lim_{x\to\infty} \frac{\sin(1/x)}{1/x} = \lim_{x\to\infty} \frac{\cos(1/x)(-1/x^2)}{-1/x^2} = \lim_{x\to\infty} \cos(1/x) = \cos(0) = 1,$$

since $1/x \to 0$ as $x \to \infty$. Thus $\displaystyle\lim_{x\to\infty} x\sin(1/x) = 1$.