Quiz 2 **Solutions**, Math 111, Section 5 (Vinroot)

For each of the following, compute the limit if it exists. If it does not exist but is infinite, describe the infinite limit and explain. Show crucial steps in your calculation and show and/or explain your work clearly.

**(a):** $\displaystyle \lim_{x\to-\infty} \frac{5x^8 - 2x^4 + 1}{3x^7 + 2x^6 - 3x^3 + 2}$

**Solution:** The highest exponent of $x$ in the denominator is $x^7$, and so we divide the numerator and denominator by $x^7$ to compute the limit:

$$\lim_{x\to-\infty} \frac{5x^8 - 2x^4 + 1}{3x^7 + 2x^6 - 3x^3 + 2} \cdot \frac{1/x^7}{1/x^7} = \lim_{x\to-\infty} \frac{5x - 2/x^3 + 1/x^7}{3 + 2/x - 3/x^4 + 2/x^7}.$$

In the numerator, we know $-2/x^3 \to 0$ and $1/x^7 \to 0$ as $x \to -\infty$, while the denominator goes to 3 as $x \to -\infty$ since $2/x$, $-3/x^4$, and $2/x^7$ all go to 0 as $x \to -\infty$. So the denominator approaches 3, and all terms but the $5x$ in the numerator go to 0. Finally, since $5x \to -\infty$ as $x \to -\infty$, then we have

$$\lim_{x\to-\infty} \frac{5x^8 - 2x^4 + 1}{3x^7 + 2x^6 - 3x^3 + 2} = -\infty.$$

**(b):** $\displaystyle \lim_{x\to\infty} \frac{\sqrt[3]{x^6 - 2x + 1}}{3x^2 + 2x - 5}$

**Solution:** We use the same technique as in **(a)**, where the highest power of $x$ in the denominator is $x^2$. We divide the numerator and denominator by $x^2$, and in the numerator we use the fact that $1/x^2 = \sqrt[3]{1/x^6}$. We then have

$$\begin{aligned}
\lim_{x\to\infty} \frac{\sqrt[3]{x^6 - 2x + 1}}{3x^2 + 2x - 5} \cdot \frac{\sqrt[3]{1/x^6}}{1/x^2} &= \lim_{x\to\infty} \frac{\sqrt[3]{1 - 2/x^5 + 1/x^6}}{3 + 2/x - 5/x^2} \\
&= \frac{\sqrt[3]{1 - 2\lim_{x\to\infty}(1/x^5) + \lim_{x\to\infty}(1/x^6)}}{3 + 2\lim_{x\to\infty}(1/x) - 5\lim_{x\to\infty}(1/x^2)} \\
&= \frac{\sqrt[3]{1 - 2(0) + 0}}{3 + 2(0) - 5(0)} = \frac{1}{3}.
\end{aligned}$$