Quiz 0 **Solutions**, Math 111, Section 4 (Vinroot)

Determine if the following limits exist, and if they exist, find the values while showing steps using properties of limits. If the limit does not exist but is infinite, then describe the infinite limit and explain.

(a): $\displaystyle \lim_{x\to\pi/2^-} \frac{x-2}{\cos(x)}$

Solution: First note that $\cos(\pi/2) = 0$, so we may not substitute in $x = \pi/2$. Next, as $x \to \pi/2^-$, then $x < \pi/2$, which means $\cos(x)$ is positive and getting close to 0. Meanwhile, we have $\pi/2 < 2$, so as $x \to \pi/2^-$, we have $x-2$ is negative, and approaching the negative number $(\pi/2)-2$. Putting this together, as $x \to \pi/2^-$, the numerator approaches a negative number, while the denominator approaches 0 but remains positive. So the quotient is negative (negative over positive) as $x \to \pi/2^-$, but since the denominator approaches zero (and the numerator does not) this is an infinite limit, and moreover a negative infinite limit. That is, we have

$$\lim_{x\to\pi/2^-} \frac{x-2}{\cos(x)} = -\infty.$$

This also means the function $f(x) = (x-2)/\cos(x)$ has a vertical asymptote at $x = \pi/2$.

(b): $\displaystyle \lim_{h\to 0} \frac{\sqrt{3+h}-\sqrt{3}}{h}$

Solution: We cannot substitute in $h = 0$ since the denominator will be 0. However, we can rationalize the numerator as follows:

$$\begin{aligned}
\lim_{h\to 0} \frac{\sqrt{3+h}-\sqrt{3}}{h} &= \lim_{h\to 0} \frac{\sqrt{3+h}-\sqrt{3}}{h} \cdot \frac{\sqrt{3+h}+\sqrt{3}}{\sqrt{3+h}+\sqrt{3}} \\
&= \lim_{h\to 0} \frac{3+h-3}{h(\sqrt{3+h}+\sqrt{3})} = \lim_{h\to 0} \frac{h}{h(\sqrt{3+h}+\sqrt{3})} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{3+h}+\sqrt{3}},
\end{aligned}$$

where we can cancel then h factor since as $h \to 0$, then $h \neq 0$, so we are not dividing by 0. Finally, we may use limit laws to evaluate this limit:

$$\lim_{h\to 0} \frac{1}{\sqrt{3+h}+\sqrt{3}} = \frac{1}{\lim\limits_{h\to 0}(\sqrt{3+h}+\sqrt{3})} = \frac{1}{\sqrt{\lim\limits_{h\to 0}(3+h)}+\sqrt{3}} = \frac{1}{\sqrt{3+0}+\sqrt{3}} = \frac{1}{2\sqrt{3}}.$$

That is, we have

$$\lim_{h\to 0} \frac{\sqrt{3+h}-\sqrt{3}}{h} = \frac{1}{2\sqrt{3}}.$$