Quiz 5 **Solutions**, Math 111, Section 5 (Vinroot)

Show all steps in the following problems.

1. Compute $\dfrac{dy}{dx}$ if $y = (\tan(x))^{x^2}$.

Solution: Since there is a (non-constant) function of x is both the base and the exponent (that is, $y = g(x)^{h(x)}$ for some non-constant $g(x)$ and $h(x)$), we use logarithmic differentiation and start by taking the natural logarithm of both sides of the equation for y. Then we have, by applying a property of logarithms,

$$\ln(y) = x^2 \ln(\tan(x)).$$

Now we differentiate both sides with respect to x. Since $\frac{d}{dx}(\ln(y)) = \frac{y'}{y}$ by the chain rule, and by applying the product and chain rules to the right side, we obtain

$$\frac{y'}{y} = \frac{d}{dx}\left(x^2 \ln(\tan(x))\right) = 2x\ln(\tan(x)) + x^2\frac{\sec^2(x)}{\tan(x)}.$$

Multiplying by y on both sides gives

$$y' = \frac{dy}{dx} = y\left(2x\ln(\tan(x)) + x^2\frac{\sec^2(x)}{\tan(x)}\right) = (\tan(x))^{x^2}\left(2x\ln(\tan(x)) + x^2\frac{\sec^2(x)}{\tan(x)}\right).$$

2. Find $f'(t)$ when $f(t) = \arctan(t^7) + \ln(\cos(2t))$. What is $f'(0)$?

Solution: To compute $f'(t)$, we need to apply the chain rule, along with the derivatives of $\arctan(t)$ and $\ln(t)$. This gives

$$\begin{aligned} f'(t) &= \frac{1}{1+(t^7)^2}\cdot 7t^6 + \frac{1}{\cos(2t)}\cdot(-\sin(2t))\cdot 2 \\ &= \frac{7t^6}{1+t^{14}} - 2\tan(2t). \end{aligned}$$

Now we can compute

$$f'(0) = \frac{7(0)^6}{1+0^{14}} - 2\tan(2(0)) = -2\tan(0) = 0.$$