Quiz 6 **Solutions**, Math 111, Section 2 (Vinroot)

As always, explain your answers (with a sentence or two) and show all steps. In both parts, let $f(x) = \frac{1}{2}x - \sqrt{x}$.

(a): Find all critical numbers of f.

Solution: The critical numbers of f are those numbers c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ is undefined. The domain of f is the set of x such that $x \geq 0$, since $\sqrt{x}$ must be defined. Next, $f'(x) = \frac{1}{2} - \frac{1}{2}x^{-1/2}$. Since $f'(0)$ is undefined, 0 is a critical number of f. To find other critical numbers, we solve $f'(c) = 0$. This occurs when

$$\begin{aligned} \frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{c}} &= 0 \\ \frac{1}{2} &= \frac{1}{2}\frac{1}{\sqrt{c}} \\ 1 &= \frac{1}{\sqrt{c}} \\ \sqrt{c} &= 1, \end{aligned}$$

which implies $c = 1$. Thus, the only critical numbers of f are 0 and 1.

(b): Find the absolute maximum and absolute minimum values of f on the interval $[0, 9]$.

Solution: To find the absolute maximum and minimum values of f on $[0, 9]$, we must evaluate f at the critical numbers in the interval, and at the endpoints of the interval. Since the critical numbers are 0 and 1 from **(a)**, we must evaluate $f(0)$, $f(1)$, and $f(9)$, and the largest and smallest values of these are the absolute maximum and minimum, respectively, of f in the interval $[0, 9]$. We have

$$f(0) = 0, \quad f(1) = \frac{1}{2} - 1 = -\frac{1}{2}, \quad f(9) = \frac{9}{2} - 3 = \frac{3}{2}.$$

So, the absolute maximum value of f on $[0, 9]$ is $\frac{3}{2}$, occurring at at $x = 9$, and the absolute minimum value of f on $[0, 9]$ is $-\frac{1}{2}$, occurring at $x = 1$.