Final Review Problems - Math 103

1. (a) $2x^2+5x-12 = (2x-3)(x+4) = 0$ when $2x-3=0$ or $x+4=0$, so $\boxed{x = \frac{3}{2} \text{ or } -4}$

(b) $x^2+3x-3=0$, and to complete the square we add and subtract $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ to the expression. Then:

$$x^2+3x+\frac{9}{4}-\frac{9}{4}-3 = \left(x+\frac{3}{2}\right)^2-\frac{9}{4}-3=0, \text{ so}$$

$$\left(x+\frac{3}{2}\right)^2 = \frac{9}{4}+3 = \frac{21}{4}, \text{ so } x+\frac{3}{2} = \pm\sqrt{\frac{21}{4}} = \frac{\pm\sqrt{21}}{2}. \text{ Then}$$

$$\boxed{x = \frac{-3}{2} \pm \frac{\sqrt{21}}{2} = \frac{-3\pm\sqrt{21}}{2}},$$

(c) From the quadratic formula for $x^2+3x+3=0$, with $a=1$, $b=3$, $c=3$, we have $b^2-4ac = 3^2-4(1)(3) = 9-12 = -3 < 0$. Then $\sqrt{b^2-4ac}$ is not a real number, so the equation has $\boxed{\text{no real solutions}}$.

(d) We can use synthetic division to find one root of $2x^3-x^2-2x+1=0$, by looking at factors of the constant term. Trying $x-1$ as a factor gives:

$$\begin{array}{r|rrrr} 1 & 2 & -1 & -2 & 1 \\ & & 2 & 1 & -1 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$$

This says $x-1$ is a factor, and we obtain $2x^3-x^2-2x+1 = (x-1)(2x^2+x-1)$

quotient is $2x^2+x-1$ remainder $=0$

1. (d) (cont'd). Factoring the quadratic, we have:

$(x-1)(2x^2+x-1) = (x-1)(2x-1)(x+1) = 0$, so

$\boxed{x = 1, \frac{1}{2}, \text{ or } -1}$.

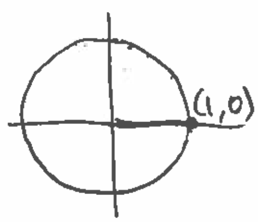
(e) [There was a typo on this problem which I have fixed]

$\cos^2(2x) - 2\cos(2x) + 1 = 0$. Letting $w = \cos(2x)$,

$w^2 - 2w + 1 = 0$ can be factored as $(w-1)^2 = 0$,

so $(\cos(2x) - 1)^2 = 0$, so $\cos(2x) - 1 = 0$, or

$\cos(2x) = 1$. For angles $2x$ in one revolution of the unit circle, $0 \leq 2x < 2\pi$, we have $\cos(2x) = 1$ when $2x = 0$.



So all angles $2x$ satisfying $\cos(2x) = 1$ are given by $2x = 0 + 2\pi k = 2\pi k$, for any integer k.

So solutions to our equation are given by

$2x = 2\pi k \Rightarrow$ $\boxed{x = \pi k, \text{ for any integer } k}$.

That is, $\boxed{x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \ldots}$

1. (f) Using properties of logarithms, we can rewrite the equation as $\ln\left(\frac{x^2}{x+3}\right) = \ln(2)$. To get rid of ln, we raise e to the power on each side, so $e^{\ln\left(\frac{x^2}{x+3}\right)} = \frac{x^2}{x+3} = e^{\ln(2)} = 2$. Now,

$\frac{x^2}{x+3} = 2$, so $x^2 = 2(x+3) = 2x+6$, so $x^2-2x-6=0$

~~Factoring gives~~ The quadratic formula, with $a=1, b=-2, c=-6$ has $b^2-4ac = (-2)^2 - 4(-6)(1) = 28 \geq 0$, so roots are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

We need to check that the original equation still makes sense with these values, since $\ln(y)$ is undefined if $y \leq 0$. We know $\ln(x^2)$ is fine, since $x^2 \geq 0$, and $1 \pm \sqrt{7} \neq 0$. For $\ln(x+3)$ and $x = 1+\sqrt{7}$, we have $x+3 = 4+\sqrt{7} > 0$, which is fine, and for $x = 1-\sqrt{7}$, $x+3 = 4-\sqrt{7} > 0$ since $4 > \sqrt{7}$ $(16 > 7)$.

So our solutions are $\boxed{x = 1+\sqrt{7}, 1-\sqrt{7}}$

1. (g) $(2e^x + A)\cos(2yb) - e^x \sin(Aby) = ze^x - \ln(\arctan(2zb))$

We first multiply out the first term (since one of these terms will have an x in it), then move all summands with e^x in it on one side, and everything else on the other side:

$$2e^x\cos(2yb) + A\cos(2yb) - e^x\sin(Aby) = ze^x - \ln(\arctan(2zb))$$

$$2e^x\cos(2yb) - e^x\sin(Aby) - ze^x = -A\cos(2yb) - \ln(\arctan(2zb)).$$

Now factor out an e^x from the left side, and divide both sides by the other terms factored out:

$$e^x\left(2\cos(2yb) - \sin(Aby) - z\right) = -A\cos(2yb) - \ln(\arctan(2zb))$$

$$e^x = \frac{-A\cos(2yb) - \ln(\arctan(2zb))}{2\cos(2yb) - \sin(Aby) - z}$$

Finally, take ln of both sides, since $\ln(e^x) = x$. So:

$$\boxed{x = \ln\left(\frac{-A\cos(2yb) - \ln(\arctan(2zb))}{2\cos(2yb) - \sin(Aby) - z}\right)}$$

We cannot further simplify this expression.

2 (a) Since $\frac{1}{8} = 2^{-3} = \left(4^{\frac{1}{2}}\right)^{-3} = 4^{\frac{-3}{2}}$, then $\log_4\left(\frac{1}{8}\right) = \boxed{\frac{-3}{2}}$.

(b) $27^{-2/3} = \left(27^{1/3}\right)^{-2} = \left(\sqrt[3]{27}\right)^{-2} = 3^{-2} = \boxed{\frac{1}{9}}$

(c) $13^{\frac{\log_7(3)}{\log_7(13)}} = 13^{\log_{13} 3}$ (since $\frac{\log_7(3)}{\log_7(13)} = \log_{13} 3$)

$= \boxed{3}$ (since $b^{\log_b y} = y$).

(d) $\ln\left(\left(e^2 e^{1/2}\right)^{-2/3}\right) = \ln\left(\left(e^{\frac{5}{2}}\right)^{\frac{-2}{3}}\right) = \ln\left(e^{-5/3}\right) =$

$= \log_e\left(e^{-5/3}\right) = \boxed{\frac{-5}{3}}$

(e) $3^{-1/2}\left(\frac{1}{27}\right)^{-1/6} = 3^{-1/2}\left(3^{-3}\right)^{-1/6} = 3^{-1/2}\, 3^{\frac{3}{6}} = 3^{-1/2}\, 3^{1/2} = 3^{-\frac{1}{2}+\frac{1}{2}} =$

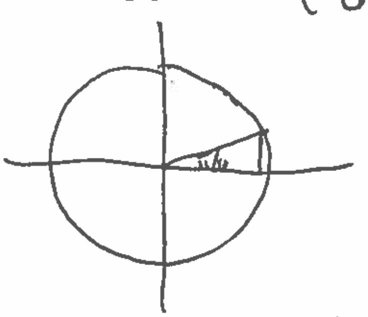
$= 3^0 = \boxed{1}$

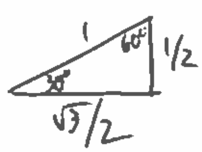
since $\ln 1 = 0$ since $34^0 = 1$

(f) $3^{\log_{28}\left(34^{\ln 1}\right)} = 3^{\log_{28}\left(34^0\right)} = 3^{\log_{28}(1)} = 3^0 = \boxed{1}$

(g) $\tan\left(\frac{13\pi}{6}\right) = \frac{\sin(13\pi/6)}{\cos(13\pi/6)}$, and $\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$.

So $\sin\left(\frac{13\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right)$ and $\cos\left(\frac{13\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right)$.



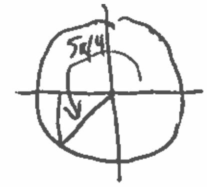


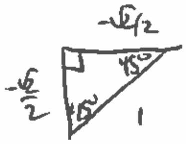
$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$,

so $\tan\left(\frac{13\pi}{6}\right) = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan\left(\frac{13\pi}{6}\right) = \boxed{\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}}$

2. (h) $\csc\left(\frac{5\pi}{4}\right) = \frac{1}{\sin\left(\frac{5\pi}{4}\right)}$, $\frac{5\pi}{4} = \pi + \frac{\pi}{4}$



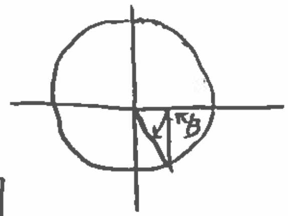


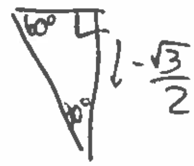
$\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, so

$$\csc\left(\frac{5\pi}{4}\right) = \frac{1}{-\sqrt{2}/2} = \frac{-2}{\sqrt{2}} = \boxed{-\sqrt{2}}$$

(i) $\sin\left(\frac{-7\pi}{3}\right)$ $\frac{-7\pi}{3} = -\left(2\pi + \frac{\pi}{3}\right) = -2\pi - \frac{\pi}{3}$, so

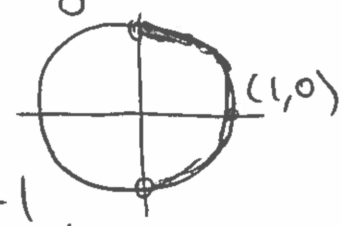
$\sin\left(\frac{-7\pi}{3}\right) = \sin\left(\frac{-\pi}{3}\right)$





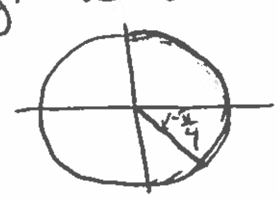
So $\sin\left(\frac{-7\pi}{3}\right) = \boxed{\frac{-\sqrt{3}}{2}}$

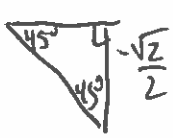
(j) $\arctan(0)$: We want an angle α such that $\tan(\alpha) = 0$ and $\frac{-\pi}{2} < \alpha < \frac{\pi}{2}$



Since $\sin(0) = 0$ and $\cos(0) = 1$, $\tan(0) = \frac{0}{1} = 0$, and $\frac{-\pi}{2} < 0 < \frac{\pi}{2}$, so $\arctan(0) = \boxed{0}$

(k) $\arcsin\left(\frac{-\sqrt{2}}{2}\right)$: We want an angle α such that $\sin(\alpha) = -\frac{\sqrt{2}}{2}$ and $\frac{-\pi}{2} \le \alpha \le \frac{\pi}{2}$

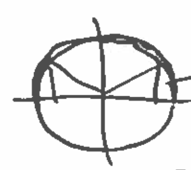




We can see from the unit circle than $\sin\left(\frac{-\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, and since $\frac{-\pi}{2} \le \frac{-\pi}{4} \le \frac{\pi}{2}$, then $\arcsin\left(\frac{-\sqrt{2}}{2}\right) = \boxed{\frac{-\pi}{4}}$

2. (d): $\sin\left(\arccos\left(\frac{1}{6}\right)\right)$. If $\arccos\left(\frac{1}{6}\right) = \theta$, then $\cos(\theta) = \frac{1}{6}$ and $0 \le \theta \le \pi$. Now we want $\sin\left(\arccos\left(\frac{1}{6}\right)\right) = \sin(\theta)$, where $\cos(\theta) = \frac{1}{6}$.

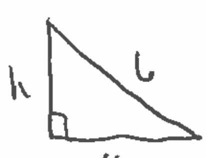
Since $\sin^2(\theta) + \cos^2(\theta) = 1$, $\sin^2(\theta) = 1 - \cos^2(\theta) = 1 - \left(\frac{1}{6}\right)^2$
$= 1 - \frac{1}{36} = \frac{35}{36}$. Since $0 \le \theta \le \pi$, we know $\sin(\theta) \ge 0$ from the unit circle:



So $\sin(\theta) = +\sqrt{\frac{35}{36}} = \frac{\sqrt{35}}{6}$. So $\boxed{\sin\left(\arccos\left(\frac{1}{6}\right)\right) = \frac{\sqrt{35}}{6}}$

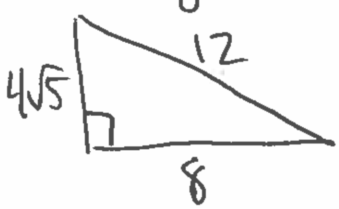
3. (a) If the second cone has height h and base radius r, the ~~second~~ first cone has height $\frac{1}{5}h$ and base radius $4r$. The ratio of the volume of the first (V_1) to the volume of the second (V_2) is then
$$\frac{V_1}{V_2} = \frac{\cancel{\frac{1}{3}}\cancel{\pi}(4r)^2\cancel{h}/5}{\cancel{\frac{1}{3}}\cancel{\pi}r^2\cancel{h}} = \frac{16\cancel{r^2}/5}{\cancel{r^2}} = \boxed{\frac{16}{5} \text{ or } 16:5}$$

(b) We can find the length of the other side of the first triangle using the Pythagorean Theorem:



$h^2 + 4^2 = 6^2$, so $h^2 + 16 = 36$, so
$h^2 = 36 - 16 = 20$, so $h = \sqrt{20} = 2\sqrt{5}$.

3 (b) (cont'd) The second triangle (the larger one) is similar to the second with hypotenuse 12. The hypotenuse of the first is 6, and $\frac{12}{6} = 2$, so the other sides of the larger triangle are twice the length of the other sides of the first:



This triangle has base 8 and height $4\sqrt{5}$, so the area is $A = \frac{1}{2} 8 (4\sqrt{5}) = \boxed{16\sqrt{5}}$

4. (a) $\dfrac{\cot^2(\alpha)}{\csc(\alpha)} = \dfrac{\frac{\cos^2(\alpha)}{\sin^2(\alpha)}}{\frac{1}{\sin(\alpha)}} = \dfrac{\cos^2(\alpha)}{\sin^{\cancel{2}}(\alpha)} \cdot \cancel{\sin(\alpha)} = \dfrac{\cos^2(\alpha)}{\sin(\alpha)}$

Since $\cos^2(\alpha) + \sin^2(\alpha) = 1$, then $\cos^2(\alpha) = 1 - \sin^2(\alpha)$.

So $\dfrac{\cos^2(\alpha)}{\sin(\alpha)} = \dfrac{1 - \sin^2(\alpha)}{\sin(\alpha)} = \dfrac{1}{\sin(\alpha)} - \dfrac{\sin^{\cancel{2}}(\alpha)}{\cancel{\sin(\alpha)}} = \csc(\alpha) - \sin(\alpha)$

So $\boxed{\dfrac{\cot^2(\alpha)}{\csc(\alpha)} = \csc(\alpha) - \sin(\alpha)}$

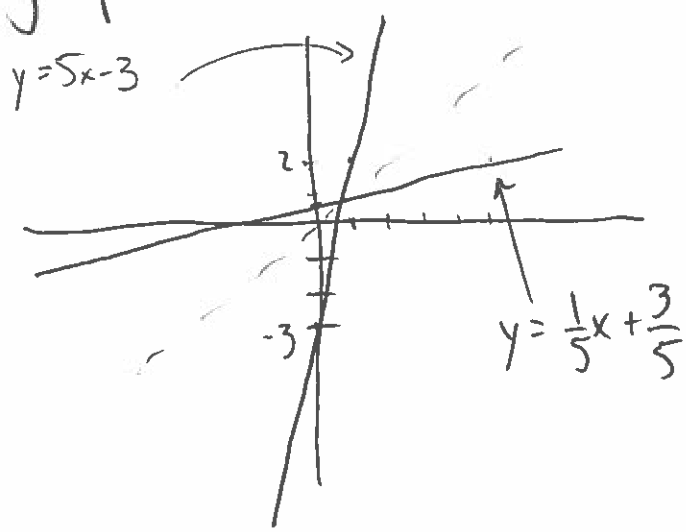
(b) Since $\cot(x) = \dfrac{1}{\tan(x)}$, $\cot^2(x)\tan^2(x) = 1$, and since $\csc(x) = \dfrac{1}{\sin(x)}$, then $\csc(x)\sin(x) = 1$. So we have

$$\underbrace{(\cot^2(x)\tan^2(x))}_{1}\sin^2(x) + (\cos(x)\underbrace{(\csc(x)\sin(x))}_{1})^2 =$$

$$= \sin^2(x) + (\cos(x))^2 = \sin^2(x) + \cos^2(x) = 1.$$

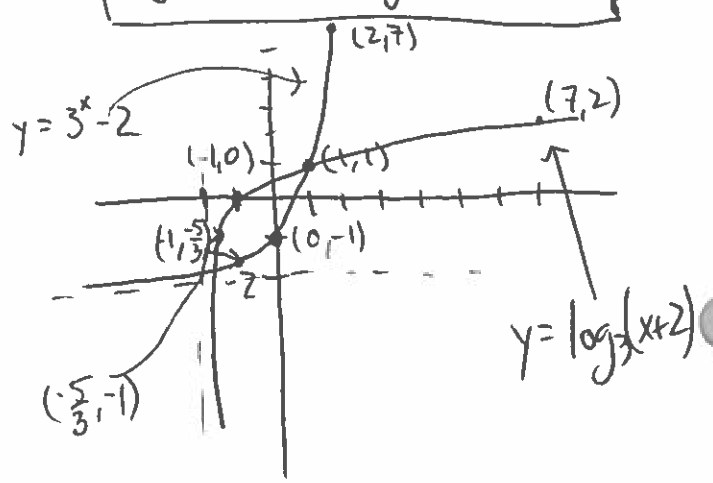
4 (b) (cont'd) So $\boxed{\cot^2(x)\tan^2(x)\sin^2(x) + (\cos(x)\csc(x)\sin(x))^2 = 1}$

5. (a) If $f(x) = 5x-3$, so $y = 5x-3$, the inverse is given by $x = 5y-3$, so $x+3 = 5y$, so $y = \frac{1}{5}x + \frac{3}{5}$. So $\boxed{f^{-1}(x) = \frac{1}{5}x + \frac{3}{5}}$. The graphs of both of these lines are given below:



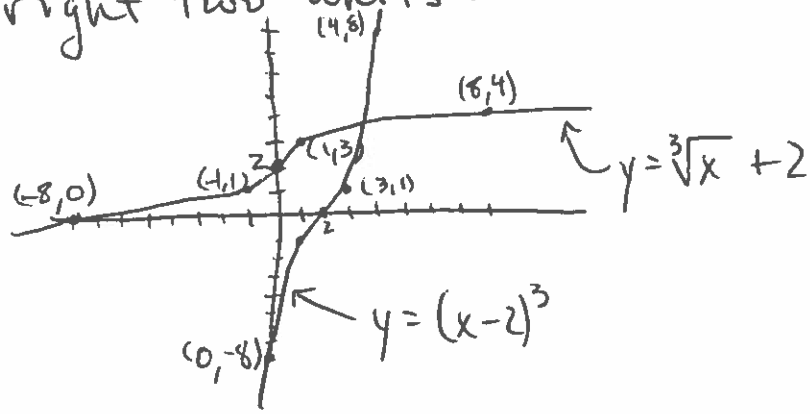
(b) If $g(x) = 3^x - 2$, so $y = 3^x - 2$, the inverse is given by $x = 3^y - 2$, so $x+2 = 3^y$, so $\log_3(x+2) = y$. So the inverse function is $\boxed{g^{-1}(x) = \log_3(x+2)}$

The graph of $y = 3^x - 2$ can be obtained by shifting $y = 3^x$ 2 units down. Points are labeled, and $y = \log_3(x+2)$ can be graphed by exchanging the x and y points:



5. (c) $h(x)=\sqrt[3]{x}+2$, so $y=\sqrt[3]{x}+2$, and the inverse function is given by $x=\sqrt[3]{y}+2$, so $x-2=\sqrt[3]{y}$, so $y=(x-2)^3$. That is, $\boxed{h^{-1}(x)=(x-2)^3}$

We can graph $y=\sqrt[3]{x}+2$ by shifting $y=\sqrt[3]{x}$ up two units, and we can graph $y=(x-2)^3$ by shifting $y=x^3$ to the right two units:



6. (a) $f(x)=\sqrt{x}-8$ has y-intercept given by the y-value when $x=0$, so $y=\sqrt{0}-8=\boxed{-8}$. The x-intercepts occurs when $y=0$, so when $0=\sqrt{x}-8$, so $\sqrt{x}=8$, so $\boxed{x=64}$. The domain is the set of all x such that the function is defined since $\sqrt{x}$ is only defined when $x\geq 0$, then $f(x)=\sqrt{x}-8$ $\boxed{\text{has domain } x\geq 0}$.

The range is the set of all outputs of the function. Since $\sqrt{x}$ is defined to be non-negative, $\sqrt{x}\geq 0$. So $f(x)=\sqrt{x}-8\geq -8$. Any output $y\geq -8$ can be obtained, since if $y\geq -8$, then $f((y+8)^2)=\sqrt{(y+8)^2}-8=y+8-8=y$.

6 (a) (cont'd) So the $\boxed{\text{range of the function is all } y \geq -8}$

(b) $g(x) = \log_2(3x+4)$ has y-intercept $g(0) =$
$= \log_2(3\cdot 0+4) = \log_2(4) = 2$. So the $\boxed{\text{y-intercept is } 2}$

The x-intercept occurs when $g(x) = \log_2(3x+4) = 0$, and raising 2 to the power on each side gives
$2^{\log_2(3x+4)} = 3x+4 = 2^0 = 1$. So $3x+4=1$, so $3x=-3$,
so $\boxed{x = -1 \text{ is the x-intercept}}$

Since $\log_b y$ is only defined when $y > 0$, the domain is the set of x such that $3x+4>0$, so $3x > -4$, so $x > \frac{-4}{3}$. So the $\boxed{\text{domain of } g \text{ is the set of all } x > \frac{-4}{3}}$.

The output of $g(x) = \log_2(3x+4)$ can be any real number y. This can be seen from either the graph, or observing that for any real number y, since $\log_2(2^y) = y$ then $g\left(\frac{2^y}{3} - \frac{4}{3}\right) = \log_2\left(3\left(\frac{2^y}{3} - \frac{4}{3}\right) + 4\right) = \log_2(2^y - 4 + 4)$
$= \log_2(2^y) = y$. That is, $\boxed{\text{the range of } g \text{ is the set of all real numbers.}}$

7. (a) $\log_7(\sqrt{x-5})$ is undefined if $\sqrt{x-5} \leq 0$ (since then the logarithm is undefined) or when $x-5<0$ (since then the square root is undefined).

We have $\sqrt{x-5} \leq 0$ when $\sqrt{x-5}=0$, since the square root is defined to be non-negative. Now $\sqrt{x-5}=0$ when $x-5=0$, or when $x=5$. Then $\sqrt{x-5}$ is undefined when $x-5<0$, so when $x<5$. Thus $\log_7\sqrt{x-5}$ is undefined when $x=5$ or $x<5$, so when $\boxed{x \leq 5}$

(b) $\dfrac{17x^{10}-13x^9+1}{\cos^2(x)}$ is undefined exactly when the denominator is 0, so when $\cos^2(x)=0$, or when $\cos(x)=0$. When $0 \leq x < 2\pi$, $\cos(x)=0$ when $x=\frac{\pi}{2}$ or $\frac{3\pi}{2}$. So the set of all x such that $\cos(x)=0$, so where the expression is undefined, is given by $\boxed{x=\frac{\pi}{2}+2\pi k \text{ or } \frac{3\pi}{2}+2\pi k\text{, for any integer } k.}$

(c) $(x^2-3x-10)^{1/4}$ is undefined when $x^2-3x-10<0$, since $\sqrt[4]{x^2-3x-10}$, an even root, is only defined for non-negative numbers. Now $x^2-3x-10<0$ when $(x-5)(x+2)<0$, 7 (c) (cont'd) so when $x-5<0$ and $x+2>0$, or, $x-5>0$ and $x+2<0$. This happens when $x<5$ and $x>-2$, or, $x>5$ and $x<-2$. But we cannot have $x>5$ and $x<-2$ so this only occurs when $x<5$ and $x>-2$, so $-2<x<5$.

So the expression is undefined when $\boxed{-2<x<5}$

(d) $\dfrac{17x^{10}-13x^9+2}{(x^2-4)(x^2+2x+5)}$ is undefined when the denominator is 0, so when $(x^2-4)(x^2+2x+5)=(x-2)(x+2)(x^2+2x+5)=0$

This happens when $x=2$, $x=-2$, or $x^2+2x+5=0$.

From the quadratic formula with $a=1$, $b=2$, $c=5$, $b^2-4ac=2^2-4(1)(5)=4-20=-16<0$, so $x^2+2x+5=0$ has no real solutions. So the expression is only undefined when $\boxed{x=2 \text{ or } -2}$.

8 (a) We use long division for polynomials:

$$\begin{array}{r}
x^2+x+2 \leftarrow \text{quotient} \\
x^2-x+2\,\big)\,\overline{x^4+0x^3+3x^2-x+2} \\
-(x^4-x^3+2x^2) \\
\hline
x^3+x^2-x+2 \\
-(x^3-x^2+2x) \\
\hline
2x^2-3x+2 \\
-(2x^2-2x+4) \\
\hline
-x-2 \leftarrow \text{remainder}
\end{array}$$

So $\dfrac{x^4+3x^2-x+2}{x^2-x+2} = x^2+x+2+\dfrac{-x-2}{x^2-x+2}$

8. (b) First, since $\frac{\log_b x}{\log_b y} = \log_y x$, we have $\frac{\ln(x^2)}{\ln(3)} = \frac{\log_e(x^2)}{\log_e(3)} =$

$= \log_3(x^2)$. So we have

$$2\log_3(x) - \log_3(x+3) - \frac{\ln(x^2)}{\ln(3)} = 2\log_3(x) - \log_3(x+3) - \log_3(x^2)$$

since $\longrightarrow$

$c\log_b(x) = \log_b(x^c)$

$$= \cancel{\log_3(x^2)} - \log_3(x+3) - \cancel{\log_3(x^2)}$$

$\longrightarrow$

$$= -\log_3(x+3) = \log_3((x+3)^{-1})$$

$$= \boxed{\log_3\left(\frac{1}{x+3}\right)}.$$

(c) We re-write $\tan(x) = \frac{\sin(x)}{\cos(x)}$, $\cot(x) = \frac{\cos(x)}{\sin(x)}$,

$\sec(x) = \frac{1}{\cos(x)}$, and $\csc(x) = \frac{1}{\sin(x)}$, and cancel things:

$$\frac{\tan(x)\sin^2(x)\cot^2(x)\sec^2(x)}{\csc^2(x)\tan^2(x)\cos^2(x)} = \frac{\frac{\sin(x)}{\cos(x)} \cdot \cancel{\sin^2(x)} \cdot \frac{\cancel{\cos^2(x)}}{\cancel{\sin^2(x)}} \cdot \frac{1}{\cancel{\cos^2(x)}}}{\frac{1}{\cancel{\sin^2(x)}} \cdot \frac{\cancel{\sin^2(x)}}{\cancel{\cos^2(x)}} \cdot \cancel{\cos^2(x)}} =$$

$$= \frac{\sin(x)}{\cos(x)} = \boxed{\tan(x)}$$

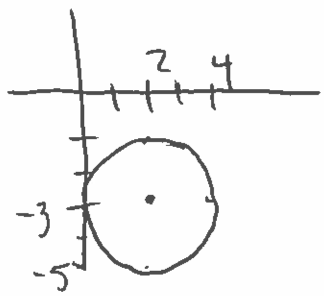
(d) We have $\frac{(x^2+2)^4}{\sqrt[3]{x^2+2}} \cdot \sqrt{(x^2+2)^{-2}}\ (x^2+2)^{2/3}$

$$= (x^2+2)^4 \cdot (x^2+2)^{-1/3} \left((x^2+2)^{-2}\right)^{1/2} (x^2+2)^{2/3}$$

8 (d) (cont'd) $=(x^2+2)^4(x^2+2)^{-1/3}(x^2+2)^{-1}(x^2+2)^{2/3}$

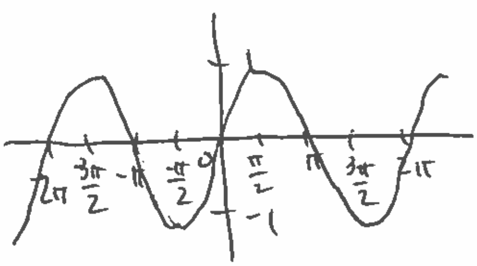
$=(x^2+2)^{4-\frac{1}{3}-1+\frac{2}{3}} = (x^2+2)^{3+\frac{1}{3}} = \boxed{(x^2+2)^{10/3}}$

9 (a) $(x-2)^2+(y+3)^2=4=2^2$ is the equation of a circle with center $(2,-3)$ and radius 2. The graph is as follows:

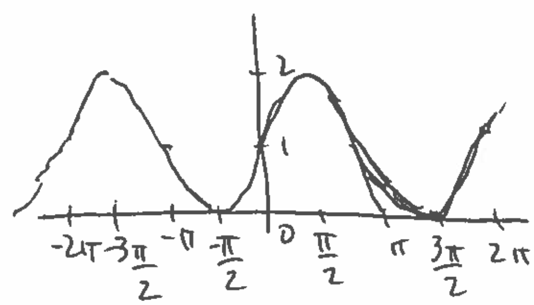


(b) $y=\sin(x+\frac{\pi}{2})+1$ can be graphed by first shifting the graph of $y=\sin(x)$ up 1 unit (to graph $y=\sin(x)+1$) then to the left $\frac{\pi}{2}$ units:

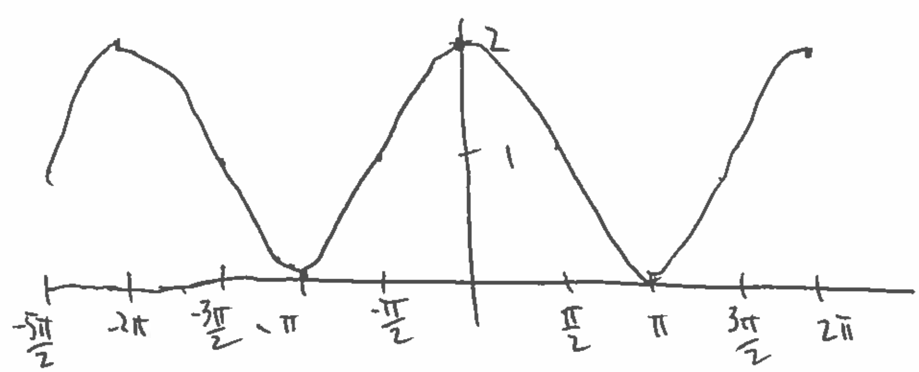
$y=\sin(x)$



$y=\sin(x)+1$

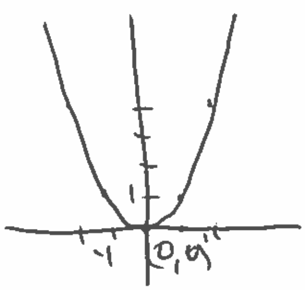


$y=\sin(x+\frac{\pi}{2})+1$:

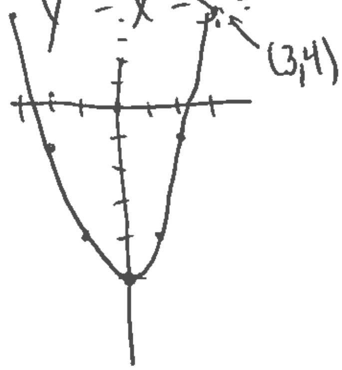


9 (c) To graph $y = |x^2 - 5|$, we first graph $y = x^2 - 5$ by shifting $y = x^2$, 5 units down. Then, $y = |x^2 - 5|$ will have the graph obtained by making all of the negative outputs of $y = x^2 - 5$ positive:

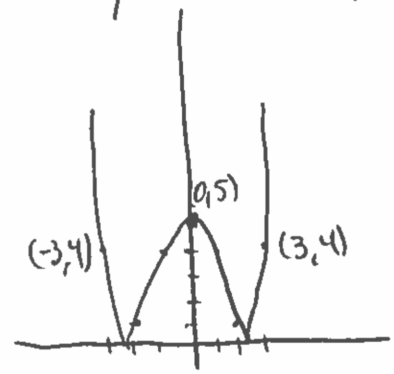
$y = x^2$:



$y = x^2 - 5$:



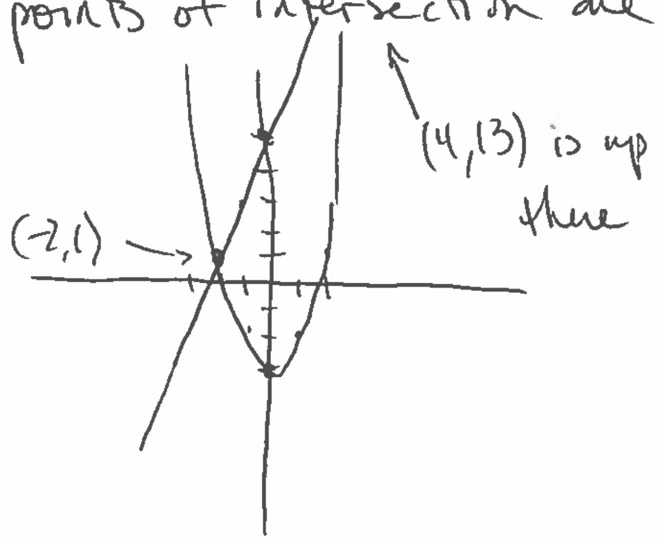
$y = |x^2 - 5|$



10 (a) The points of intersection of $y = x^2 - 3$ and $y = 2x + 5$ occur when $x^2 - 3 = 2x + 5$, so when $x^2 - 2x - 8 = 0$, so when $(x-4)(x+2) = 0$, or when $x = 4$ or $x = -2$.

The y-values are given by $y = 2(4) + 5 = 13$ and $y = 2(-2) + 5 = 1$. So the points of intersection are $(4,13)$ and $(-2,1)$.

The graphs of these are as follows:



10 (b) The point of intersection occurs when $\frac{-1}{\ln(x)} = \ln(x) - 2$,

so when $-1 = (\ln(x))^2 - 2(\ln(x))$, so when

$(\ln(x))^2 - 2\ln(x) + 1 = 0$. Letting $w = \ln(x)$, we can factor

this as $w^2 - 2w + 1 = (w-1)^2 = (\ln(x) - 1)^2 = 0$.

So $\ln(x) - 1 = 0$, so $\ln(x) = 1$. Raising e to the

power on each side, $e^{\ln(x)} = e^1$, so $x = e$.

The y-value is given by $\ln(e) - 2 = 1 - 2 = -1$, so the

point of intersection is $\boxed{(e, -1)}$

11. First note that if $f(x) = x^3$, the inverse function is

$f^{-1}(x) = x^{1/3} = \sqrt[3]{x}$. Also, if $g(2) = -1$ (and g has

an inverse function, which I have corrected by

stating this now), then $g^{-1}(-1) = 2$. So:

(a) $f(g(2)) = f(-1) = (-1)^3 = \boxed{-1}$

(b) $f^{-1}(-8) = (-8)^{1/3} = \boxed{-2}$

(c) $f(g^{-1}(-1)) = f(2) = 2^3 = \boxed{8}$

(d) $f(f^{-1}(213)) = f(\sqrt[3]{213}) = (\sqrt[3]{213})^3 = \boxed{213}$

12 (a) If $y = \log_3(x^3-5)$, the inverse is given by $x = \log_3(y^3-5)$. Raising 3 to the power on each side gives $3^x = 3^{\log_3(y^3-5)} = y^3-5$. Then $y^3 = 3^x+5$, and taking the cube root gives $y = (3^x+5)^{1/3}$. So $\boxed{f^{-1}(x) = (3^x+5)^{1/3}}$

(b) If $y = \frac{1}{e^{x^5}+2}$, then the inverse is given by

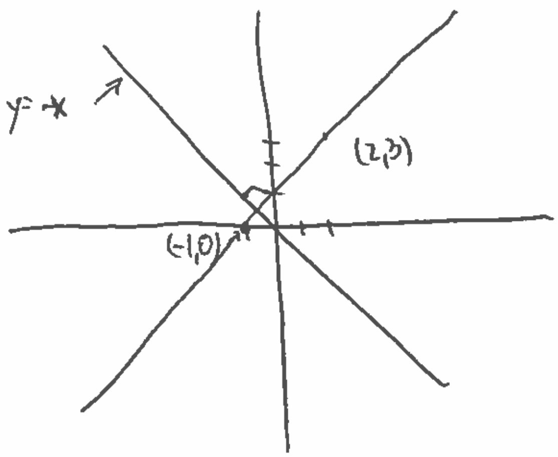
$x = \frac{1}{e^{y^5}+2}$. Taking the reciprocal of both sides, $e^{y^5}+2 = \frac{1}{x}$, so $e^{y^5} = \frac{1}{x}-2$. Now take ln of both sides, so

$\ln(e^{y^5}) = y^5 = \ln\left(\frac{1}{x}-2\right)$. Finally, take the fifth root of both sides, so $y = \left(\ln\left(\frac{1}{x}-2\right)\right)^{1/5}$. So $\boxed{f^{-1}(x) = \left(\ln\left(\frac{1}{x}-2\right)\right)^{1/5}}$.

13 (a) The (second) line, through the points $(2,3)$ and $(-1,0)$, has slope $\frac{3-0}{2-(-1)} = \frac{3}{3} = 1$. So a line perpendicular to it has slope $\frac{-1}{1} = -1$. The equation of the line through $(1,-1)$ and perpendicular to the first line has equation $y = -x+b$, and with $x=1$, $y=-1$,

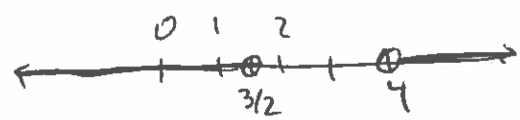
13 (a) (cont'd) we have $-1=-1+b$, so $b=0$.

So the line has equation $\boxed{y=-x}$. The graphs of the two lines are below:

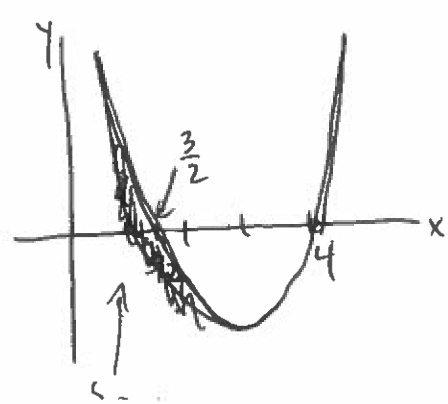


(b) We have $2x^2-11x+12>0$ when $(2x-3)(x-4)>0$, so when $2x-3>0$ and $x-4>0$, or, $2x-3<0$ and $x-4<0$, so when $x>\frac{3}{2}$ and $x>4$, or, $x<\frac{3}{2}$ and $x<4$.

Now $x>\frac{3}{2}$ and $x>4$ means $x>4$, while $x<\frac{3}{2}$ and $x<4$ means $x<\frac{3}{2}$. So $2x^2-11x+12>0$ when $x>4$ or $x<\frac{3}{2}$



From factoring, $2x^2-11x+12=0$ when $x=4$ or $x=\frac{3}{2}$, and so $2x^2-11x+12<0$ for all other values, when $\frac{3}{2}<x<4$. These tell us when the graph of $y=2x^2-11x+12$ is below or above the x-axis:



14 (a) $f(x)=\sqrt{x^2+2}$ could only be undefined if the square root is taken of a negative number. However, $x^2 \geq 0$ for any real x, so $x^2+2 \geq 2$, and $\sqrt{x^2+2}$ is defined for every real x. So $\boxed{\text{the domain of } f \text{ is all real numbers}}$

The range is the set of all outputs, and since the square root is non-negative, no negative output can occur. Also, as we saw above, $x^2+2 \geq 2$, which means $\sqrt{x^2+2} \geq \sqrt{2}$. We can obtain any $y \geq \sqrt{2}$ as an output, since if $y \geq \sqrt{2}$, then $f(\sqrt{y^2-2}) = \sqrt{(\sqrt{y^2-2})^2+2} = \sqrt{y^2-2+2} = \sqrt{y^2}$ $= y$. So $\boxed{\text{the range is all } y \geq \sqrt{2}}$

Since $\cos(x)$ is defined for any input x, the so is $g(x)=\cos(x^2)$. So $\boxed{\text{the domain of } g \text{ is all real numbers}}$

The output of $\cos(x)$ is the interval $-1 \leq y \leq 1$, from the unit circle definition of $\cos(x)$. Then $g(x)=\cos(x^2)$ has the same range, since, for example, $g(0)=\cos(0^2)=\cos(0)=1$, and $g(\sqrt{\pi})=\cos((\sqrt{\pi})^2)=\cos(\pi)=-1$

If $\cos(t)=y$, where $-1 \leq y \leq 1$, then $g(\sqrt{t})=\cos((\sqrt{t})^2)=$ $=\cos(t)=y$. That is, the range of g is all y such that $\boxed{-1 \leq y \leq 1}$

14 (b) $f(g(x)) = f(\cos(x^2)) = \sqrt{(\cos(x^2))^2+2} = \boxed{\sqrt{\cos^2(x^2)+2}}$

$g(f(x)) = g(\sqrt{x^2+2}) = \cos((\sqrt{x^2+2})^2) = \boxed{\cos(x^2+2)}$

(c) Neither $f(x)$ nor $g(x)$ have inverse functions. We can see this by checking that they both fail the "horizontal line test", not by graphing, but rather by seeing two different input values give the same output. For $f(x)$, we have $f(1) = \sqrt{1^2+2} = \sqrt{3}$, while $f(-1) = \sqrt{(-1)^2+2} = \sqrt{3} = f(1)$. So, the horizontal line $y=\sqrt{3}$ will intersect the graph of $y=f(x)$ in the points $(1,\sqrt{3})$ and $(-1,\sqrt{3})$.

For $g(x)$, $g(0) = \cos(0^2) = \cos(0) = 1$, and $g(\sqrt{2\pi}) = \cos((\sqrt{2\pi})^2) = \cos(2\pi) = 1$. So, the horizontal line $y=1$ will intersect the graph of $y=\cos(x^2)$ at the points $(0,1)$ and $(\sqrt{2\pi},1)$ (and at infinitely many more points $(\sqrt{2\pi k},1)$, for k a positive integer).