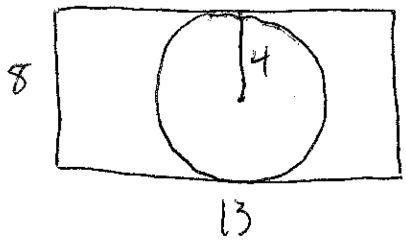
Math 103 HW #1

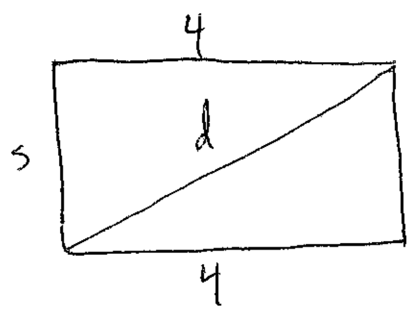
A1.



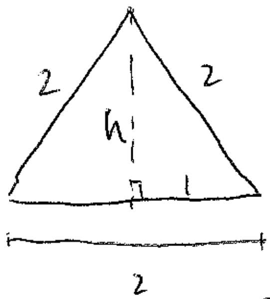
Since the shortest side of the rectangle is 8, the maximum diameter of a circle inside of it is 8. The maximum radius is thus $\frac{1}{2}(8) = 4$.

A2. The total angle degree sum for a convex pentagon is $180n - 360 = 180(n-2)$ for $n = 5$, so $180(3) = 540$. Three angles sum to $100° + 90° + 110° = 300°$, so the sum of the remaining two is $540° - 300° = 240°$. Their average measure is then $\frac{1}{2}(240°) = 120°$.

A3.

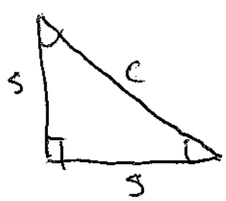


The perimeter of the rectangle is 14, and if the other side is length s, then $4+4+s+s = 8+2s$ is also the perimeter. So $8+2s = 14$, $2s = 6$, so $s = 3$. The area is then $3 \cdot 4 = 12$, and if the diagonal is d, then $3^2 + 4^2 = d^2$, so $9 + 16 = 25 = d^2$, and $d = \sqrt{25} = 5$.

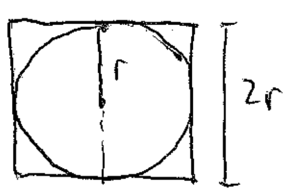
A4. 

Taking a height perpendicular to one base, this splits the base into segments of length 1.

Applying the Pythagorean Theorem, $1^2 + h^2 = 2^2$, so $1 + h^2 = 4$, $h^2 = 3$, and $h = \sqrt{3}$.

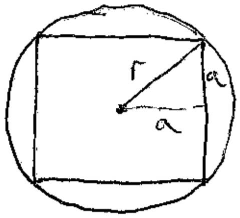
A5. 

The non-right angles are equal in measure, and sum to $90°$, so they each must be $45°$. Since the non-hypotenuse sides have the same length, if the hypotenuse has length c, then $s^2 + s^2 = c^2$, so $2s^2 = c^2$, and $\sqrt{2s^2} = \sqrt{c^2}$, so $c = s\sqrt{2}$.

A6. 

If the radius of the circle is r, then the side of the square has length $2r$.

So, ~~ex~~ the area of the square is $(2r)^2 = 4r^2$, and the area of the circle is πr^2, and the ratio asked for is $\frac{\pi r^2}{4r^2} = \frac{\pi}{4}$ or $\pi : 4$.

A7. 

If the radius of the circle is r, its area is πr^2 and its circumference is $2\pi r$. To find these for the square, we need the side length of the square. If we draw the right triangle with hypotenuse r as in the picture, the other two sides have the same length, say a, where $2a$ is the side length of the square. From the Pythagorean Theorem,

$$a^2 + a^2 = r^2, \text{ so } 2a^2 = r^2, \text{ so } a\sqrt{2} = r, \text{ or } a = \frac{r}{\sqrt{2}}.$$

The area of the square is now $(2a)^2 = \left(\frac{2r}{\sqrt{2}}\right)^2 =$

$= \frac{4r^2}{2} = 2r^2$, so the ratio of the areas is

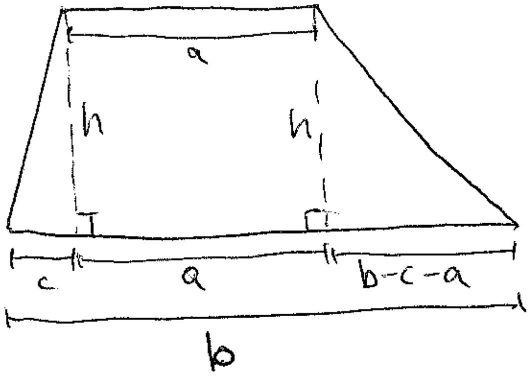
$\frac{\pi r^2}{2r^2} = \frac{\pi}{2}$ or $\pi : 2$. The perimeter of the square is

$4\left(\frac{2r}{\sqrt{2}}\right)$. Note $\frac{2r}{\sqrt{2}} = \frac{(\sqrt{2})^2 r}{\sqrt{2}} = \sqrt{2}\, r$, so

the perimeter is $4\left(\frac{2r}{\sqrt{2}}\right) = 4\sqrt{2}\, r$. The ratio of the circumference to the perimeter is now

$$\frac{2\pi r}{4\sqrt{2}\, r} = \frac{\pi}{2\sqrt{2}} \text{ or } \pi : 2\sqrt{2}.$$

A8.



(i) The segment on the bottom edge between the two heights has length a, since it must be the same as the top base edge. Since the total length of the bottom base is b, and two segments are length c and a, the third must be $b-c-a$ (since $c+a+(b-c-a)=b$).

(ii) The left triangle has base c and height h, so has area $\frac{1}{2}ch$. The triangle on the right has base $b-c-a$ and height h, so has area $\frac{1}{2}(b-c-a)h$.

(iii) The area of the trapezoid is the sum of the areas of the two triangles and the a-by-h rectangle in the middle. That rectangle has area ah, so the total area is

$$\frac{1}{2}ch + \frac{1}{2}(b-c-a)h + ah = \frac{1}{2}ch + \frac{1}{2}(bh-ch-ah)+ah$$

$$= \cancel{\frac{1}{2}ch} + \frac{1}{2}bh - \cancel{\frac{1}{2}ch} - \frac{1}{2}ah + ah$$

$$= \frac{1}{2}bh - \frac{1}{2}ah + ah = \frac{1}{2}bh + \frac{1}{2}ah = \boxed{\frac{1}{2}(b+a)h}$$