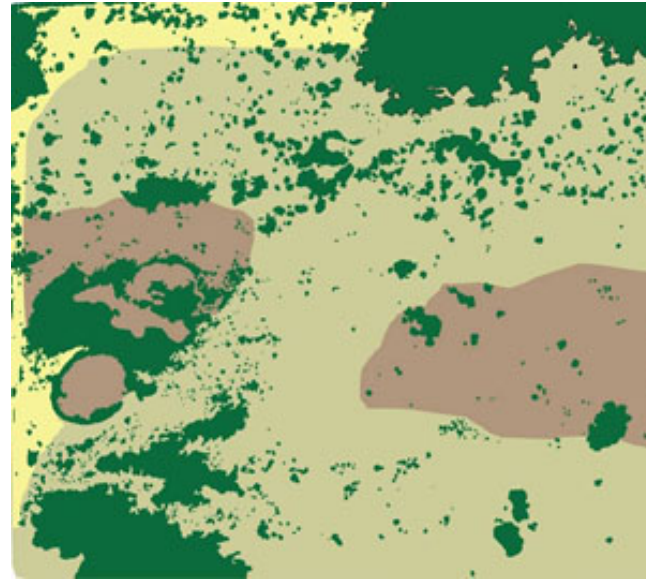
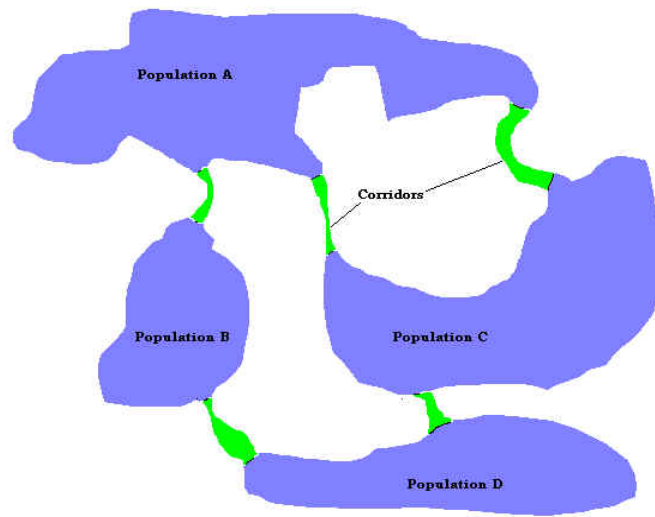


Metapopulation: population of population

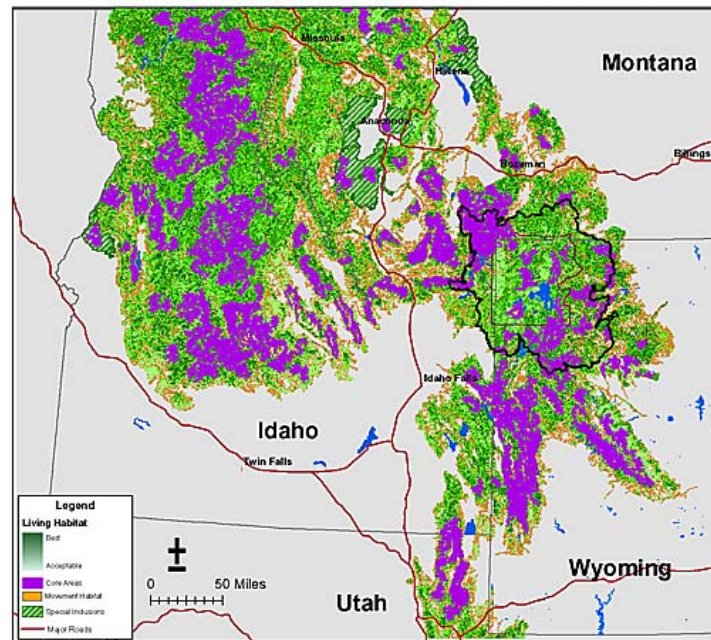
Population lives in habitats. But habitats are often fragmented by the roads, shops, factories, etc. Thus the population lives in smaller habitats, which we call patches. Population on each patch is a local population, and the totality of all local population is a metapopulation.

Each local population is governed by a growth pattern of itself, and some patches are connected so population can move from one patch to another so emigration occurs through corridors between the patches.

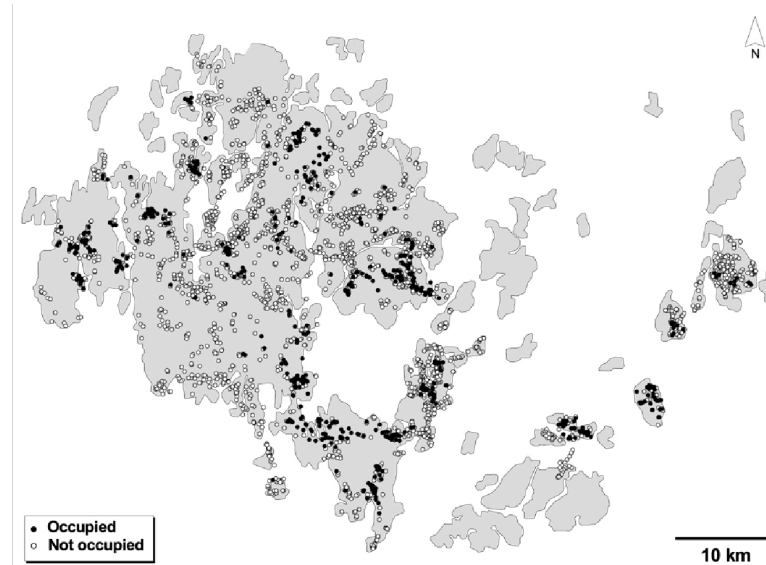


Metapopulation: patches

Regional Scale Grizzly Bear
Combined Analysis for the Central Rockies



patches of habitat for grizzle bears in central Rocky mountain



Metapopulation of Glanville fritillary butterflies (Finland)
http://www.helsinki.fi/science/metapop/english/Research/Project_metapop.htm

Simplest and earliest model: Levins model (1969)

Levins, R. (1969) "Some demographic and genetic consequences of environmental heterogeneity for biological control." Bulletin of the Entomology Society of America, 71, 237-240

There are many patches, and the state of the patch is either occupied by the species, or not occupied (so local dynamics is ignored).

(a) Let $p(t)$ be the fraction of sites which are occupied at time t . So $0 \leq p(t) \leq 1$.

(b) Let c be the rate at which the colonists are produced if all sites are produced. So the rate at which colonists are produced is cp .

(c) The fraction of unoccupied sites is $1 - p$; thus the rate of colonization of unoccupied sites is $cp(1 - p)$.

(d) Let e be the rate of local extinctions on occupied sites.

Levins model:
$$\frac{dp}{dt} = cp(1 - p) - ep = (c - e)p - cp^2$$

mathematically this is logistic model!

If $c > e$, all solutions tend to equilibrium $p_* = 1 - (e/c)$;

If $c \leq e$, all solutions tend to equilibrium $p_0 = 0$.

So to avoid the extinction of metapopulation, the colonization rate has to be greater than extinction rate.

Matrix: numbers in a table (spreadsheet)
(A vector is $1 \times n$ matrix or $n \times 1$ matrix)

A 2×2 matrix: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ A 2×3 matrix: $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$

$\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix}$ can be written as $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$

Matrices multiplication

Multiplying a $k \times n$ matrix and $n \times m$ matrix, you get a $k \times m$ matrix. Number of columns of the first matrix is the same as the number of rows of the second matrix. (You can multiply a 3×4 matrix and 4×2 matrix, but not a 4×2 matrix and a 3×4 matrix.)

Data fitting for Logistic model: (estimate r and K , P_0 is known)

Notice that $\ln \frac{P}{K - P} = \ln \frac{P_0}{K - P_0} + rt$

(a) Get a set of data: $(t_1, P_1), (t_2, P_2), \dots, (t_n, P_n)$.

(b) Let $Q_i = \ln(P_i) - \ln(K - P_i)$, and get new data set $(t_1, Q_1), (t_2, Q_2), \dots, (t_n, Q_n)$.

(c) Put the data set (t_i, Q_i) to your linear regression program and get the output slope k and intercept b .

(d) r is the maximum growth rate per capita in the Logistic equation. $b = \ln \frac{P_0}{K - P_0}$, from which we can solve K provided P_0 is known.