Assignment 3

Math 345, Prof. Shi

Due: Wednesday , Sept 27 (11am)

1. **Determining a clearance rate** Consider a patient receiving a drug intravenously at a rate of 10 mg/hour. An hour later the concentration of drug in the patients body is 1 mg/liter. Assuming the patient has 5 liters of blood and the drug is lost at a rate proportional to amount of drug in the body, (a) Find the clearance rate of the drug, and (b) determine the limiting concentration of drug in the patients body.

2. Suppose that $N(t)$ denotes the size of a population at time t. The population evolves according to the logistic equation but, in addition, predation reduces the size of the population so that the rate of change is given by $\dfrac{dN}{dt} = N\left(1 - \dfrac{N}{50}\right) - \dfrac{9N}{5+N}$.
   (a) Find the equilibrium points of the equation.
   (b) Determine the stability of the equilibrium points by using linearization or phase line.
   (c) Sketch the phase line. If $N(0) = 35$, what is the limit of $N(t)$ as $t \to \infty$?

3. Consider the logistic equation $N' = rN\left(1 - \dfrac{N}{K(t)}\right)$ where the carrying capacity $K(t)$ is time-dependent. In each of the following cases, describe the long term behavior of $N(t)$, using Matlab program `dfield`.
   (a) Assume $K(t) = 1 - (1/2)e^{-t}$, $r = 1$. This might model a human population where, due to technological improvement, the availability of resources is increasing with time, although ultimately limited.
   (b) Assume $K(t) = 1 - (1/2)\cos(2\pi t)$, $r = 1$. Here the carrying capacity is periodic in time with period 1. This models, for example, a population of insects or small animals affected by the seasons.

4. Consider a population model: $\dfrac{dx}{dt} = kx\left(1 - \dfrac{x}{N}\right) - M$, $x(0) = x_0$, where k, M, N, x_0 are positive parameters.
   (a) In the following table, fill in the dimensions of all parameters in terms of the dimensions of variables.

Variable	Dimension	Parameter	Dimension
t	τ	k	
x	λ	M	
		N	
		x_0	

   (b) Use the change of variable:
$$y = \frac{x}{N}, \quad s = kt.$$
   Derive the new equation (including the initial condition) in the new variables y and s.

5. If the equation in problem 1 is now
$$\frac{dN}{dt} = N\left(1 - \frac{N}{50}\right) - \frac{aN}{5+N},$$
where $a > 0$ is the predation rate. When $a > a_0$, the population will become extinct no matter how large the initial value is. Determine this a_0.