Abstracts of talks in International Workshop on Banach Spaces, Operator Theory and Applications to Nonlinear Analysis Harbin Normal University, Harbin, China

Some Geometric Properties of Tensor Products

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Abstract: It is known that the projective tensor product $X\hat{\otimes}_\pi Y$ and the injective tensor product $X\check{\otimes}_\varepsilon Y$ of Banach lattices X and Y may not be Banach lattices, while the Fremlin projective tensor product $X\hat{\otimes}_{|\pi|}Y$ and the Wittstock injective tensor product $X\check{\otimes}_{|\varepsilon|}Y$ of Banach lattices X and Y are always Banach lattices. In this talk, we will discuss under what circumstances $X\hat{\otimes}_\pi Y$ and $X\check{\otimes}_\varepsilon Y$ are Banach lattices, and under what circumstances some geometric properties can be inherited from two Banach spaces X and Y to their tensor products $X\hat{\otimes}_\pi Y$ and $X\check{\otimes}_\varepsilon Y$, and from two Banach lattices X and Y to their tensor products $X\hat{\otimes}_{|\pi|}Y$ and $X\check{\otimes}_{|\varepsilon|}Y$.

Extension of maps on operator spaces and completely rank nonincreasing linear maps

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Abstract: We talk about the question that under what conditions a (linear or nonlinear) map acting on a subspace (or a subset) has an extension to a Jordan homomorphisms. We show that the completely rank non-increasing linear maps acting on nest algebras or on subspaces of finite von Neumann algebras have the desired property.

Orlicz spaces without extreme points

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Abstract:

Quasi-approximate Equivalence and Essential Equivalence of Operators

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Abstract: Let $\mathcal{H}$ be a separable Hilbert space, and $\mathcal{B}(\mathcal{H})$ the algebra of bounded linear operators on $\mathcal{H}$. For A in $\mathcal{B}(\mathcal{H})$, we denote the adjoint operator of A by A^*. Say that A is essentially normal if $A^*A - AA^*$ is compact. In this talk, *quasi-approximate $(\mathcal{U}+\mathcal{K})$-equivalence*, an equivalence relation for essentially normal operators, will be introduced. And, with respect to this equivalence, a canonical form of essentially normal operators will be given. Moreover, we also investigate the essential similarity and the strongly approximate similarity of operators.

On ψ-direct sums of Banach spaces

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Abstract: Based on the result in Bonsall and Duncan [1] (see [3]) concerning the correspondence between an absolute normalized norm on $\mathbf{C}^2$ and a convex function on the unit interval, the notion of ψ-direct sum $X \oplus_\psi Y$ of Banach spaces X and Y was introduced in [5] (see [2] for finitely many Banach spaces; cf. also [4]). This extends the notion of ℓ_p-sum $X \oplus_p Y$ and has been attracting a good deal of attention; various interesting results were obtained by several authors.

As the ψ-norm on the direct sum $X \oplus Y$ is constructed from a convex function ψ on the unit interval, we can construct many concrete norms, especially those of non-ℓ_p-type, on the direct sum $X \oplus Y$. Thus it is quite natural to ask how various properties of $X \oplus_\psi Y$ are described by means of the convex function ψ and those of X and Y.

In this lecture we shall introduce this notion from the origin so as to be understandable for all students and present some recent results, which will serve as a preliminary to my talk in the Workshop.

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Some recent results on direct sums of Banach spaces

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Abstract: Let ψ be a convex (continuous) function on the unit interval satisfying the conditions

$$\psi(0) = \psi(1) = 1 \text{ and } \max\{1-t, t\} \le \psi(t) \le 1 \text{ for all } 0 \le t \le 1. \tag{1}$$

The *ψ-direct sum* $X \oplus_\psi Y$ of Banach spaces X and Y is the direct sum $X \oplus Y$ equipped with the norm

$$\|(x,y)\|_\psi = \begin{cases} (\|x\| + \|y\|)\psi\left(\frac{\|y\|}{\|x\|+\|y\|}\right) & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0) \end{cases} \tag{2}$$

([5]; see [1] for finitely many Banach spaces). This extends the notion of ℓ_p-sum $X \oplus_p Y$ and the ℓ_1- and ℓ_∞-norms are the largest and the smallest such norms respectively: $\|\cdot\|_\infty \leq \|\cdot\|_\psi \leq \|\cdot\|_1$.

Thus from convex functions ψ on the unit interval we can construct many concrete norms, especially those of non-ℓ_p-type, on the direct sum $X \oplus Y$. Our problem is to ask how various properties of $X \oplus_\psi Y$ are described by means of ψ and those of X and Y.

We shall present some recent results on the ψ-direct sum, especailly concerning the uniform non-ℓ_1^n-ness ([2,3,4]). The ℓ_1- and ℓ_∞-sums will be also discussed as the extreme cases.

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Existence of type(II) region and convexity and concavity of potential functional corresponding jumping nonlinear probelms

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Abstract:

Weak Orlicz space and its applications to martingale theory

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Abstract: The weak Orlicz space is introduced and its applications to martingale theory are discussed. In particular, some martingale inequalities in weak Orlicz spaces and relationships between these martingale spaces are established, and the boundedness of some martingale operators are investigated, their vector-valued analogues are also considered.

Imperfect transcritical and pitchfork bifurcations

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Abstract: We consider the solution set of $F(\varepsilon, \lambda, u) = 0$, where X and Y are Banach spaces, and $F : \mathbf{R} \times \mathbf{R} \times X \to Y$ is a differentiable nonlinear mapping. Here the parameter ε indicates the perturbation. Following [S1, S2], we investigate the imperfect bifurcation of bifurcation diagram in (λ, u) space under small perturbations. Imperfection of the bifurcation diagrams occur when the small errors or noises destroy the original bifurcation structure, which occur frequently in engineering or other application problems. We prove several new theorems about the symmetry breaking of transcritical and pitchfork bifurcations. Applications include semilinear elliptic equations, imperfect Euler buckling beam problem and perturbed diffusive logistic equation. The talk is based on joint work with Junping Shi and Yuwen Wang [LSW1,LSW2].

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Three Classes of Smooth Banach Submanifolds in $B(E, F)$

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Abstract: Let E, F be two Banach spaces, and let $B(E, F), \Phi(E, F), S\Phi(E, F)$ and $R(E, F)$ be the bounded linear, Fredholm, semi-Fredholm and finite rank operators from E into F, respectively. In this paper, using continuity characteristics of generalized inverses of operators under small perturbations, we prove the following result: Let Σ be any one of the following sets: $\{T \in \Phi(E, F) : \text{Index}T = \text{const. and dim } N(T) = \text{const.}\}$, $\{T \in S\Phi(E, F) :$ either $\dim N(T)$ =constant$< \infty$ or codim $R(T)$ =constant$< \infty\}$ and $\{T \in R(E, F) :$ Rank$T$ =constant$< \infty\}$. Then Σ is a smooth submanifold of $B(E, F)$ with the tangent space $T_A\Sigma = \{B \in B(E, F) : BN(A) \subset R(A)\}$ for any $A \in \Sigma$. The result is available for further application to Thom's famous results on the transversility and the study of infinite dimensional geometry.

Interpolation between Lp and Lq spaces

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Type, cotype and convexity properties of quasi-Banach spaces

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Abstract: The notions of type and cotype of Banach spaces emerged from the work of J. Hoffman-Jörgensen, S. Kwapien, B. Maurey, B. Maurey and G. Pisier in early 1970's. Since then, these notions have found frequent use in geometry of Banach spaces as well as in probability theory in Banach spaces. For the Banach lattices a related notions of p-convexity, q-concavity and upper p-estimate, lower q-estimate appeared in Krivine paper and then in classical Lindenstrauss-Tzafriri book [6].

These concepts are natural in quasi-Banach spaces where we have triangle inequality with some constant $C \geq 1$ instead of $C = 1$ or in p-Banach spaces, $0 < p < 1$ (cf. [4]). Many definitions and classical results in Banach spaces can be translated in a natural way to definitions and results in quasi-Banach or p-Banach spaces. These was done in eighties by N. J. Kalton ([2, 3]). We study properties and results on type, cotype, convexity and concavity of quasi-Banach spaces. Moreover, we will show how to use some techniques for the classical Lorentz spaces with arbitrary weight which was obtained in the paper [5] (cf. also [8]).

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Nonlinear Ill-posed Problems

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Slant Differentiability and Semismooth Methods for Operator Equations

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Abstract: Let X and Y be Banach spaces, and $F : D \subset X \to Y$ be a continuous mapping on an open domain D. The following concepts of slant differentiability and slanting function were introduced in *X. Chen, Z. Nashed, and L.Qi, smoothing methods and semismooth methods for nondifferentiable operator equations, SIAM J. Numer. Anal. 38 (2000), no. 4, 1200-1216. A function $F : D \subset X \to Y$ is said to be* **slantly differentiable at** $x \in D$ *if there exist a mapping $f^\circ : D \to L(X,Y)$ such that the family $f^\circ(x+h)$ of bounded linear operators is uniformly bounded in the operator norm for h sufficiently small and*

$$\lim_{h\to 0} \frac{F(x+h) - F(x) - f^0(x+h)h}{\| h \|} = 0.$$

The function f° is called a **slanting function** *for F* **at** *x. A function $F : D \subset X \to Y$ is said to be slantly differentiable in an open domain $D_0 \subset D$ if there exists a mapping $f^0 : D \to L(X,Y)$ such that f° is a slanting function for F at every point $x \in D_0$. In this case, f° is called a* **slanting function** *for F* **in** *D_0.*

In this talk I will discuss operator-theoretic aspects of slant differentiability and related variants. Application to Newton-like methods, optimal control theory, and nonlinear ill-posed problems will be indicated. A unifying framework for semismooth analysis will be sketched and compared with the setting in *M. Z. Nashed, Differentiability and related*

properties of nonlinear operators: Some aspects of the role of differentials in nonlinear functional analysis, in L.B. Rall, ed., "Nonlinear Functional Analysis and Applications", Academic Press, New York, 1971, pp. 103-309. for smooth analysis.

Extensions of bifurcation from simple eigenvalue theorem

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Abstract: Nonlinear equations can often be formulated as a nonlinear operator equation in Banach spaces, and the linearized operators are often linear Fredholm operators of index zero. Examples include a large class of quasilinear elliptic systems with nonlinear boundary conditions. A classical theorem of Crandall and Rabinowitz states that a bifurcation occurs at a simple eigenvalue along a branch of trivial solutions, and near the bifurcation point, the solution set consists of a line segment and a curve crossing it. We show a bifurcation theorem generalizing the classical one without *a priori* knowledge of a line of trivial solutions, which can also be used in perturbation problem of degenerate solutions. In another extension we show that the bifurcation from a simple eigenvalue is indeed global, which is also an extension of Rabinowitz's global bifurcation theorem which requires compactness. The talk is based on joint work with Ping Liu, Yuwen Wang of Harbin Normal University, and Xuefeng Wang of Tulane University.

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Composition operators on vector-valued spaces of analytic functions

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Abstract: Let D be the unit disk in the complex plane and φ an analytic self-map of D. The theory of analytic composition operators C_φ; $f \mapsto f \circ \varphi$ on various Banach spaces consisting of scalar-valued analytic functions on D (such as e.g. the Hardy and Bergman spaces, $BMOA$) is very extensive. My talk will describe some recent work about properties of the same composition operator $f \mapsto f \circ \varphi$ on Banach spaces of vector-valued analytic functions $f : D \to X$, where X is an infinite dimensional complex Banach space (see e.g. [LST], [BDL], [LT], [L1] and [L2]). In the vector-valued setting it is natural to consider strong and weak types of spaces of analytic functions.

The topics discussed will include: (1) qualitative properties of C_φ, such as weak compactness, (2) a characterization of the boundedness of $C_\varphi : wH^p(X) \to H^p(X)$ in terms of the Hilbert-Schmidt norm of $C_\varphi : H^2 \to H^2$ for $2 \leq p < \infty$, [LTW]. (Here $wH^p(X)$ is the weak X-valued Hardy space defined by requiring that $\sup_{\|x^*\| \leq 1} \|x^* \circ f\|_{H^p} < \infty$, with $x^* \in X^*$.)

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Subgradient-Based Differential Inclusion for Nonsmooth Nonconvex Optimization Problems

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Abstract: This paper develops a neural network for solving the nonsmooth nonconvex optimization problems with a nonsmooth nonconvex objection function, a class of linear equality constraints and a class of nonsmooth convex inequality constrains. The proposed neural network is modeled by a differential inclusion. Because there is no neural network to solve this type of optimization problem, the neural network proposed in this paper is much available and very important. Under a suitable assumption on the constraint set and a proper assumption on the objective function, it is proved that for a given nonsmooth nonconvex optimization problem and a sufficiently large penalty parameter, the trajectory of the neural network can reach the feasible region in finite time and stays there thereafter. And we can prove that the trajectory of the neural network constructed by a differential inclusion and with arbitrarily given initial value belonging to any bounded set containing the feasible region S, converges to the set consisted by the equilibrium points of the neural network, which coincides with the set consisted by the critical points of the objective function over the feasible region. In particular, we give the condition ensuring convergence to the equilibrium point set in finite time. Moreover, under suitable assumptions, we prove the coincidence between the neural network modeled by a differential inclusion and the "slow solution" of it. Furthermore, illustrative examples show the correctness of the results in this paper, and the good performance of the proposed neural network.