#### CSCI 658

# DISCRETE OPTIMIZATION

Course Syllabus—Fall 2024

MWF 10-10:50 a.m. Jones Hall 113

**INSTRUCTOR:** Rex K. Kincaid Office—126 Hugh Jones Hall Email—rrkinc@wm.edu

**PREREQUISITES:** Csci 520 and Csci 628 or their equivalent (linear programming theory, rudimentary compelxity theory and C programming).

**DESCRIPTION:** Discrete optimization problems are those problems with decisions that are logical (yes/no) or countable. Both exact and heuristic methods for discrete optimization models will be presented in the course. Topics include relaxation techniques, constructive heuristics, improving search techniques (simplex method, simulated annealing, tabu search and genetic algorithms), branch and bound schemes, and valid inequalites for branch and cut methods.

**KEY DATES:** The add/drop deadline is September 9 and the withdraw deadline is October 28. No classes on September 2 (Labor Day), October 10-13 (Fall Break), November 5 (Election Day), and November 27 - December 1 (Thanksgiving). The final exam is on Tuesday, December 10 from 2-5 p.m..

## **TOPIC OUTLINE:**

- 1. Basic Discrete Optimization Methods [14 lectures] (Ra Ch.11,12, Luke Ch. 1,2)
  - (a) Enumeration and Relaxations
  - (b) Strengthening Relaxations
  - (c) Branch and Bound
  - (d) Refinements to Branch and Bound
  - (e) Gradient Based Optmization (Luke Ch.1)
  - (f) Single State Methods (Luke Ch.2)
  - (e) Improving Search Heuristics
  - (f) Constructive Heuristics
- 2. Extending Discrete Improving Search [9 lectures]
  - (a) Simulated Annealing (JAMS, Ra Ch.12)
  - (b) Tabu Search (GL, Gl, Ra Ch.12)
  - (c) Genetic Algorithms (B, G)

## Midterm Exam tentatively scheduled for October 23, 2024

- 3. Lagrangean Relaxation [8 lectures] (PR Ch.5, Be)
  - (a) Introduction
  - (b) Exposition
  - (c) Choosing Lagrange Multipliers

- 4. Matroids and Integer Solvability of ILPs [5 lectures]
  - (a) Matroids (PR Ch 3.1-3.4)
  - (b) Submodular Functions (PR 3.7)
  - (c) Unimodularity and Total Dual Integrality (PR 4.2)
- 5. Connections with Reinforcement Learning [2 lectures] (RL-TS, RL-TS2)

### Comprehensive Final Exam: Tuesday, December 10 (2-5 p.m.)

#### **GRADES:**

There will be a midterm exam and a final exam. The final exam is scheduled for Tuesday, December 10 from 2 p.m. until 5 p.m.. Each exam will count 35 % of the final course grade. The exams will be open notes. Homework assignments will be given periodically throughout the semester and together will count 30 % of the final course grade. Some homework assignments will involve C programming and the use of AMPL and linear programming software.

#### **REFERENCES:**

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- Be Beasley, J.E., "Lagrangean Relaxation," (1992) 1-37, The Management School, Imperial College, London SW7 2AZ, England.
- G Goldberg, D.E. <u>GAs in Search Optimization and Machine Learning</u>, Addison Wesley (1989).
- Gl Glover, F., "Tabu Search: A Tutorial," INTERFACES, 20 (1990) 74-94.
- G2 Glover, F., "Tabu Search-Part I," ORSA J. on Computing, 1 (1989) pp. 190-206.
- GL Glover, F. and M. Laguna, "Tabu Search," chapter in Modern Heuristics for Combinatorial Problems, (1993).
- Gui Guigard, M, "Lagrangian Relaxation," Top, Volume 11, Number 2, December 2003. Published by Sociedad de Estadistica e Investigacion Operativa, Madrid, Spain
- JAMS Johnson, D.S., C.R. Aragon, L.A. McGeoch, and C. Schevon, "Opimization by Simulated Annealing: An Experimental Evaluation; Part I, Graph Partitioning," *Operations Research*, 37 (1989) 865-892.
  - PR Parker, R.G. and R.L. Rardin, "Chapter 5: Nonpolynomial Algorithms–Partial Enumeration," *Discrete Optimization*, Academic Press, Boston (1988).
  - Ra Rardin, R.L., *Modeling and Analysis in Operations Research*, Chapters 3, 11, and 12 (1994) partial manuscript.
  - Ra2 Rardin, R.L., Discrete Optimization, Chapter 106 in <u>Handbook of Industrial Engineering</u>, Wiley, NY (1992).
  - Luke Luke, S., <u>Essentials of Metaheuristics</u>, Lulu, 2nd edition (2016).
  - SK Skorin-Kapov, J., "Tabu Search Applied to the Quadratic Assignment Problem," ORSA J. on Computing, 2 (1990) pp. 33-42.
- Talbi Talbi, El-Ghazali, <u>Metaheuristics: from design to implementation</u>, Wiley (2009). See chapters 1-3.
- Talbi2 Karimi-Mamaghan, M., M. Mohammadi, P. Meyer, A.M. Karimi-Mamaghan, and E. Talbi, "Machine Learning at the Service of Meta-heuristics for solving Combniatorial Optimization Problems: A State-of-the-art," *EJOR*, Volume 296, Issue 2, 16 January 2022, Pages 393-422.
- RL-CO Mazyavkina, N., S. Sviridov, S. Ivanov, and E. Burnaev, "Reinforcement Learning for Combinatorial Optimization: A Survey," *Computers & Operations Research*. Volume 134, October 2021,
- RL-CO2 Darvariu, V-A., S. Hailes, and M. Musolesi, "Graph Reinforcement Learning for Combinatorial Optimization: A Survey and Unifying Perspective," (2024) https://arxiv.org/abs/2404.06492
  - RL-TS Li,M., J-K. Hao, and Q. Wu, "Learning-Driven Feasible and Infeasible Tabu Search for Airport Gate Assignment." *European Journal of Operational Research* Volume 302, Issue 1, 1 October 2022, Pages 172-186, DOI:10.1016/j.ejor.2021.12.019.
- RL-TS2 Sun, Z., U, Benlic, M. Li, and Q. Wu, "Reinforcement learning based tabu search for the minimum load coloring problem," *Computers & Operations Research*, Volume 143, July 2022.