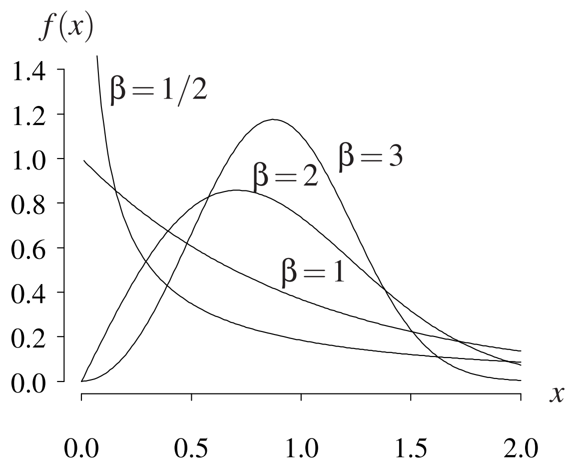
Weibull distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{Weibull}(\alpha, \beta)$ is used to indicate that the random variable X has the Weibull distribution with scale parameter $\alpha > 0$ and shape parameter $\beta > 0$. A Weibull random variable X has probability density function

$$f(x) = \frac{\beta}{\alpha} x^{\beta-1} e^{-(1/\alpha)x^{\beta}} \qquad x > 0.$$

The Weibull distribution is used in reliability and survival analysis to model the lifetime of an object, the lifetime of a organism, or a service time. The accelerated life and Cox proportional hazards model are identical when the baseline distribution is Weibull. The probability density function is plotted below for $\alpha = 1$ and $\beta = 1/2, 1, 2, 3$.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = 1 - e^{-(1/\alpha)x^{\beta}} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = e^{-(1/\alpha)x^{\beta}} \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\beta}{\alpha} x^{\beta-1} \qquad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \frac{1}{\alpha} x^{\beta} \qquad x > 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = [-\alpha \ln(1-u)]^{1/\beta} \qquad 0 < u < 1.$$

The median of X is

$$(\alpha \ln 2)^{1/\beta}.$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \int_0^\infty e^{tx} \frac{\beta}{\alpha} x^{\beta-1} e^{-(1/\alpha)x^\beta}\, dx \qquad t > 0.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = \int_0^\infty e^{itx} \frac{\beta}{\alpha} x^{\beta-1} e^{-(1/\alpha)x^\beta}\, dx \qquad t > 0.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{\alpha}{\beta}\Gamma\left(\frac{1}{\beta}\right) \qquad V[X] = \alpha^2\left\{\frac{2}{\beta}\Gamma\left(\frac{2}{\beta}\right) - \left[\frac{1}{\beta}\Gamma\left(\frac{1}{\beta}\right)\right]^2\right\}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \left\{\frac{2}{\beta}\Gamma\left(\frac{2}{\beta}\right) - \left[\frac{1}{\beta}\Gamma\left(\frac{1}{\beta}\right)\right]^2\right\}^{-3/2}\left\{\frac{3}{\beta}\Gamma\left(\frac{3}{\beta}\right) - \frac{6}{\beta^2}\Gamma\left(\frac{1}{\beta}\right)\Gamma\left(\frac{2}{\beta}\right) + 2\left[\frac{1}{\beta}\Gamma\left(\frac{1}{\beta}\right)\right]^3\right\}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \left\{\frac{2}{\beta}\Gamma\left(\frac{2}{\beta}\right) - \left[\frac{1}{\beta}\Gamma\left(\frac{1}{\beta}\right)\right]^2\right\}^{-2}\left\{\frac{4}{\beta}\Gamma\left(\frac{4}{\beta}\right) - \frac{12}{\beta^2}\Gamma\left(\frac{1}{\beta}\right)\Gamma\left(\frac{3}{\beta}\right)\right.$$
$$\left. + \frac{12}{\beta^3}\left[\Gamma\left(\frac{1}{\beta}\right)\right]^2\Gamma\left(\frac{2}{\beta}\right) - \frac{3}{\beta^4}\left[\Gamma\left(\frac{1}{\beta}\right)\right]^4\right\}.$$

APPL verification: The APPL statements

```
X := WeibullRV(1 / alpha, beta);
CDF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, hazard function, population mean, variance, skewness, kurtosis, and moment generating function.