**von Mises distribution** (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html) The shorthand  $X \sim \text{von Mises}(\kappa, \mu)$  is used to indicate that the random variable X has the von Mises distribution with shape parameter  $\kappa$  and location parameter  $\mu$ . A von Mises random variable X with parameters  $\kappa$  and  $\mu$  has probability density function

$$f(x) = \frac{e^{\kappa \cos(x-\mu)}}{2\pi I_0(\kappa)} \qquad \qquad 0 < x < 2\pi$$

for all  $\kappa$  and for  $0 < \mu < 2\pi$ . The Modified Bessel function of the first kind of order 0 is used for the von Mises random variable and is defined as

$$I_0(\kappa) = \sum_{i=0}^{\infty} \frac{\kappa^{2i}}{2^{2i}(i!)^2}$$

for all  $\kappa$ . The probability density function for  $\mu = \pi$  and two different values of  $\kappa$  is illustrated below.



The cumulative distribution function on the support of *X* is

$$F(x) = P(X \le x) = \frac{\int_0^x e^{\kappa \cos(t-\mu)} dt}{2\pi I_0(\kappa)} \qquad \qquad 0 < x < 2\pi.$$

The survivor function, hazard function, inverse distribution function, moment generating function, and characteristic functions are all mathematically intractable. The median of X is

μ.

The population mean of *X* is

$$E[X] = \mu$$

The population variance, skewness, and kurtosis of *X* are mathematically intractable.