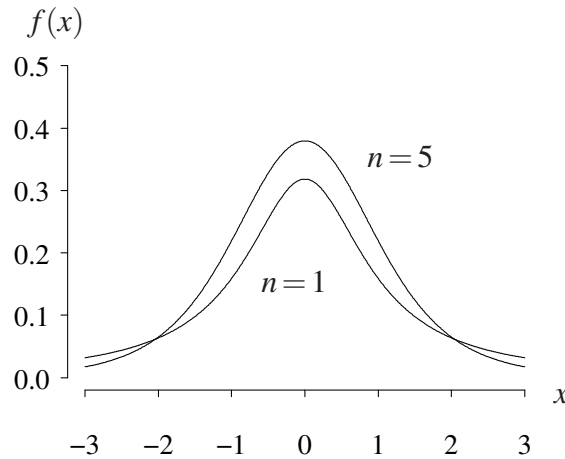**T distribution** (from)
The shorthand $X \sim t(n)$ is used to indicate that the random variable X has the t distribution with positive, real-valued parameter n, known as the degrees of freedom. In most applications, n is a positive integer. A t random variable X with n degrees of freedom has probability density function

$$f(x) = \frac{\Gamma((n+1)/2)(1+x^2/n)^{(-(n+1)/2)}}{\sqrt{n\pi}\Gamma(n/2)} \qquad -\infty < x < \infty.$$

The t distribution arises in hypothesis tests concerning the comparison of (a) a sample mean to a standard, or (b) the difference between two means. The probability density function for $n = 1$ (a special case of the t distribution known as the Cauchy distribution) and $n = 5$ is illustrated below.

The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} \frac{\Gamma((n+1)/2)n^{n/2}(n+t^2)^{((n-1)/2)}}{\sqrt{\pi}\Gamma(n/2)} \, dt \qquad -\infty < x < \infty.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \int_{x}^{\infty} \frac{\Gamma((n+1)/2)n^{n/2}(n+t^2)^{((n-1)/2)}}{\sqrt{\pi}\Gamma(n/2)} \, dt \qquad -\infty < x < \infty.$$

The hazard function and cumulative hazard function on the support of X is mathematically intractable.

The inverse distribution function of X is mathematically intractable.

The median of X is 0 because $f(x)$ is an even function.

The moment generating function of X is undefined, although various moments exist for restricted

values of the parameter n.

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = 0 \quad \text{for } n > 1, \qquad\qquad V[X] = \frac{n}{n-2} \quad \text{for } n > 2,$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \quad \text{for } n > 3, \qquad\qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3(n-2)}{n-4} \quad \text{for } n > 4.$$

APPL verification: The APPL statements

```
X := TRV(n);
CDF(X);
SF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, and kurtosis.