Theorem A standard uniform random variable X can be transformed to a log logistic random variable Y through the transformation

$$Y = \frac{1}{\lambda}\left(\frac{1-X}{X}\right)^{1/\kappa},$$

where λ and κ are positive.

Proof Let the random variable X have the standard uniform distribution with probability density function

$$f_X(x) = 1 \qquad 0 < x < 1.$$

The transformation $Y = g(X) = \frac{1}{\lambda}\left(\frac{1-X}{X}\right)^{1/\kappa}$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, 0 < x < 1\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = \frac{1}{1+(\lambda Y)^\kappa}$ and Jacobian

$$\frac{dX}{dY} = \frac{-\kappa\lambda^\kappa Y^{\kappa-1}}{[1+(\lambda Y)^\kappa]^2}.$$

Therefore by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y))\left|\frac{dx}{dy}\right| \\ &= (1)\left|\frac{-\kappa\lambda^\kappa y^{\kappa-1}}{[1+(\lambda y)^\kappa]^2}\right| \\ &= \frac{\lambda\kappa(\lambda y)^{\kappa-1}}{[1+(\lambda y)^\kappa]^2} \qquad y > 0, \end{aligned}$$

which is the probability density function of the log logistic distribution.

APPL verification: The APPL statements

```
X := StandardUniformRV();
assume(lambda > 0);
assume(kappa > 0);
g := [[x -> 1 / lambda * ((1 - x) / x) ^ (1 / kappa)], [0, 1]];
Y := Transform(X, g);
simplify(Y[1][1](y));
Z := LogLogisticRV(lambda, kappa);
```

yield equivalent the functional forms

$$f_Y(y) = \frac{\lambda\kappa(\lambda y)^{\kappa-1}}{[1+(\lambda y)^\kappa]^2} \qquad y > 0,$$

for the random variables Y and Z, which verifies that the standard uniform distribution can be transformed to the log logistic distribution.