Theorem The distribution of $\max\{X_1, X_2, \ldots, X_n\}$, where $X_1, X_2, \ldots, X_n$ are independent and identically distributed standard power(β) random variables, has the standard power distribution.

Proof The standard power distribution has probability density function

$$f(x) = \beta x^{\beta-1} \qquad\qquad 0 < x < 1$$

and cumulative distribution function

$$F(x) = x^{\beta} \qquad\qquad 0 < x < 1,$$

where β is a positive parameter. Let $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$. Using the order statistic result

$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!(n-k)!}[F(x)]^{k-1}[1-F(x)]^{n-k} f(x) \qquad\qquad a < x < b;\ k = 1, 2, \ldots, n,$$

where $f(\cdot)$ and $F(\cdot)$ denote the population probability density function and cumulative distribution function, and a and b are the minimum and maximum of the population support. Setting $k = n$, the probability density function of $X_{(n)}$ is

$$f_{X_{(n)}}(x) = n x^{(n-1)\beta} \beta x^{\beta-1} = n\beta x^{n\beta-1} \qquad\qquad 0 < x < 1.$$

This can be recognized as a random variable having a standard power distribution with parameter $n\beta$.

APPL verification: The APPL statements

```
assume(beta > 0);
assume(n, posint);
X := [[x -> x ^ beta], [0, 1], ["Continuous", "CDF"]];
Y := OrderStat(X, n, n);
simplify(Y[1][1](x));
```

yield the probability density function of a standard power random variable with parameter $n\beta$.