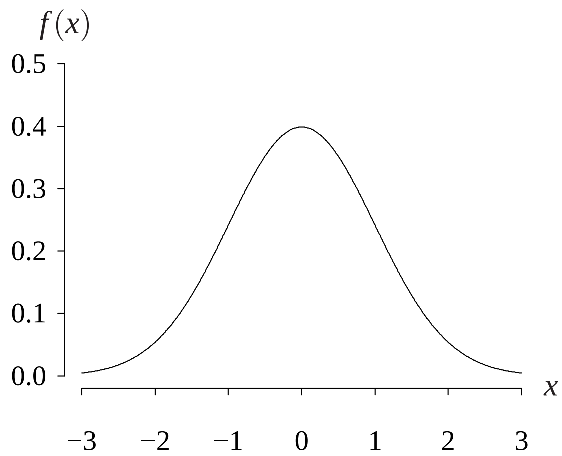
Standard normal distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim N(0,1)$ is used to indicate that the random variable X has the standard normal distribution. A standard normal random variable X has probability density function

$$f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \qquad -\infty < x < \infty.$$

The standard normal random variable arises because a normal random variable with mean μ and variance σ^2 can be standardized by subtracting μ, then dividing by σ. This means that only a single table is required for all calculations involving the normal distribution. The probability density function is illustrated below.



The cumulative distribution function is

$$F(x) = \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right) \qquad -\infty < x < \infty$$

where

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt \qquad x > 0,$$

and $\mathrm{erf}(-x) = -\mathrm{erf}(x)$. The survivor function on the support of X is

$$S(x) = \frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right) \qquad -\infty < x < \infty.$$

The hazard function on the support of X is

$$h(x) = -\frac{\mathrm{e}^{-\frac{1}{2}x^2}\sqrt{2}}{\sqrt{\pi}\left(-1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right)} \qquad -\infty < x < \infty.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \ln(2) - \ln\left(1 - \text{erf}\left(\frac{x}{\sqrt{2}}\right)\right) \qquad -\infty < x < \infty.$$

The inverse distribution function of X is

$$F^{-1}(u) = \sqrt{2}(\text{erf}^{-1}(2u-1)) \qquad 0 \le u \le 1.$$

The median and mode of X are 0.

The moment generating function of X is

$$M(t) = \text{e}^{t^2/2} \qquad -\infty < t < \infty.$$

The characteristic function of X is

$$\phi(t) = \text{e}^{-t^2/2} \qquad -\infty < t < \infty.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = 0 \qquad V[X] = 1 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = 3.$$

APPL verification: The APPL statements

```
X := StandardNormalRV();
CDF(X);
SF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, inverse distribution function, population mean, variance, skewness, kurtosis, and moment generating function.