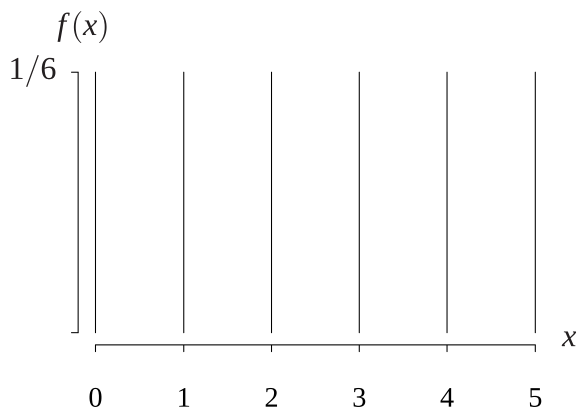
Rectangular distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{rectangular}(n)$ is used to indicate that the random variable X has the rectangular distribution with positive parameter n. A rectangular random variable X for some positive integer n has probability mass function

$$f(x) = \frac{1}{n+1} \qquad x = 0, 1, 2, \ldots, n.$$

The probability mass function with $n = 5$ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = \frac{x+1}{n+1} \qquad x = 0, 1, 2, \ldots, n.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{n+1-x}{n+1} \qquad x = 0, 1, 2, \ldots, n.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{1}{n+1-x} \qquad x = 0, 1, 2, \ldots, n.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = -\ln\left(\frac{n+1-x}{n+1}\right) \qquad x = 0, 1, 2, \ldots, n.$$

The inverse distribution function of X is

$$F^{-1}(u) = \lfloor (n+1)u \rfloor \qquad 0 < u < 1.$$

The median of X is

$$\frac{n-1}{2},$$

when n is even.

The moment generating function of X is

$$M(t) = E\left[e^{tx}\right] = \begin{cases} 1 & t = 0 \\ \frac{e^{t(n+1)}-1}{(e^t-1)(n+1)} & t \neq 0. \end{cases}$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itx}\right] = \begin{cases} 1 & t = 0 \\ \frac{e^{it(n+1)}-1}{(e^{it}-1)(n+1)} & t \neq 0. \end{cases}$$

The population mean and variance of X are

$$E[X] = \frac{n}{2} \qquad V[X] = \frac{n(n+2)}{12}.$$

The skewness and kurtosis of X are

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3}{5} \cdot \frac{3n^2+6n-4}{n(n+2)}.$$

APPL verification: The APPL statements

```
assume(n, posint);
X := [[x -> 1 / (n + 1)], [0 .. n], ["Discrete", "PDF"]];
CDF(X);
SF(X);
HF(X);
CHF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, population mean, variance, skewness, kurtosis, and moment generating function.