**Theorem** The Rayleigh distribution has the variate generation property. That is, the inverse cumulative distribution function of a Rayleigh($\alpha$) random variable can be expressed in closed-form.

**Proof** The cumulative distribution function of a Rayleigh($\alpha$) random variable $X$ is given by

$$F(x) = \int_0^x \frac{2w}{\alpha} e^{-w^2/\alpha}\, dw = 1 - e^{-x^2/\alpha} \qquad x > 0.$$

The inverse cumulative distribution function is

$$F^{-1}(u) = \sqrt{\alpha \ln\left(\frac{1}{1-u}\right)} \qquad 0 < u < 1,$$

which is closed-form. Therefore, the Rayleigh distribution has the variate generation property.

**APPL verification:** The APPL statements

```
assume(alpha > 0);
X := [[x -> 2 * x / alpha * exp(-x ^ 2 / alpha)], [0, infinity],
      ["Continuous", "PDF"]];
solve(CDF(X)[1][1](x) = u, x);
```

verify the result.