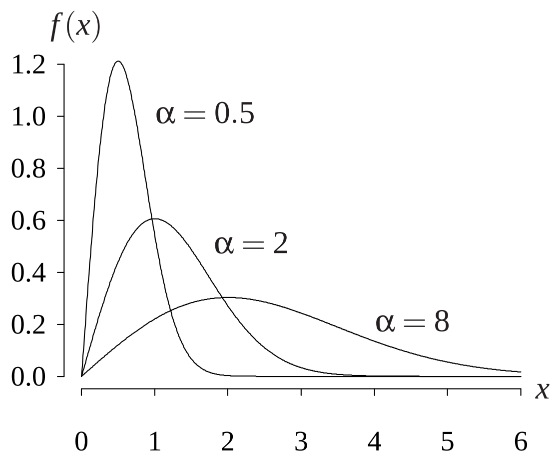
Rayleigh distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{Rayleigh}(\alpha)$ is used to indicate that the random variable X has the Rayleigh distribution with parameter α. A Rayleigh random variable X with positive parameter α has probability density function

$$f(x) = \frac{2xe^{-x^2/\alpha}}{\alpha} \qquad x > 0.$$

The Rayleigh distribution can be used to model the lifetime of an object or a service time. The probability density function with three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = 1 - e^{-x^2/\alpha} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = e^{-x^2/\alpha} \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{2x}{\alpha} \qquad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \frac{x^2}{\alpha} \qquad x > 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = \sqrt{-\alpha \ln(1-u)} \qquad 0 < u < 1.$$

The median of X is

$$\sqrt{\alpha \ln(2)}.$$

The moment generating function of X is mathematically intractable.

The characteristic function of X is mathematically intractable.

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{\sqrt{\alpha\pi}}{2} \qquad V[X] = \frac{\alpha(4-\pi)}{4} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{2\sqrt{\pi}\,(\pi-3)}{(4-\pi)^{3/2}}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{6\pi(4-\pi)-16}{(\pi-4)^2}.$$

APPL verification: The APPL statements

```
assume(alpha > 0);
X:=[[x -> (2 * x / alpha) * exp(-x ^ 2 / alpha)], [0,infinity],
    ["Continuous", "PDF"]];
CDF(X);
HF(X);
CHF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution function, hazard function, cumulative hazard function, population mean, variance, skewness, and kurtosis.