Theorem The Pascal(n, p) distribution is a special case of the power series$(c, A(c))$ distribution when $c = 1 - p$ and $A(c) = (1 - c)^{-n}$.

Proof The power series$(c, A(c))$ distribution has probability mass function

$$f(x) = \frac{a_x c^x}{A(c)} \qquad x = 0, 1, 2, \ldots.$$

When $c = 1 - p$, $A(c) = (1 - c)^{-n}$, and

$$f(x) = \frac{a_x (1-p)^x}{p^{-n}} = a_x p^n (1-p)^x \qquad x = 0, 1, 2, \ldots.$$

Setting $a_x = \binom{n+x-1}{x}$,

$$f(x) = \binom{n+x-1}{x} p^n (1-p)^x \qquad x = 0, 1, 2, \ldots,$$

which is the probability mass function of the Pascal(n, p) distribution.

APPL verification: The APPL statements

```
assume(c > 0);
assume(a[x] > 0);
X := [[x -> a[x] * c ^ x / A(c)], [0, infinity], ["Discrete", "PDF"]];
c := 1 - p;
A := c -> (1 - c) ^ (-n);
a[x] := binomial(n + x - 1, x);
simplify(X[1][1](x));
```

yield the probability mass function of a Pascal(n, p) random variable.