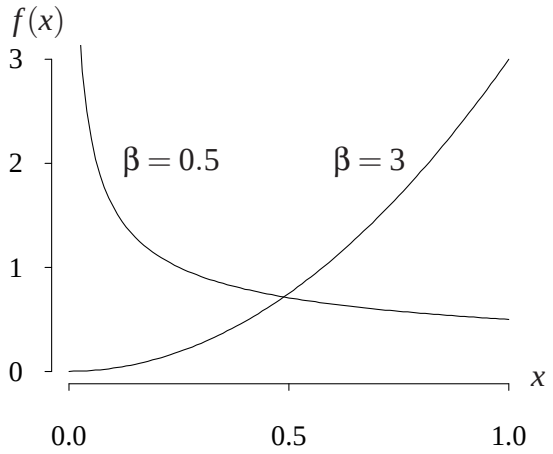
Power distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{power}(\alpha, \beta)$ is used to indicate that the random variable X has the power distribution with parameters α and β. A power random variable X with positive scale parameter α and positive shape parameter β has probability density function

$$f(x) = \frac{\beta x^{\beta-1}}{\alpha^{\beta}} \qquad 0 < x < \alpha.$$

The probability density function with $\alpha = 1$ and two different values of β is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{x^{\beta}}{\alpha^{\beta}} \qquad 0 < x < \alpha.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = 1 - \frac{x^{\beta}}{\alpha^{\beta}} \qquad 0 < x < \alpha.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\beta x^{\beta-1}\alpha^{-\beta}}{\alpha^{\beta} - x^{\beta}} \qquad 0 < x < \alpha.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = -\ln\left(1 - (x/\alpha)^{\beta}\right) \qquad 0 < x < \alpha.$$

The inverse distribution function of X is

$$F^{-1}(u) = \alpha u^{1/\beta} \qquad 0 < u < 1.$$

The median of X is

$$\alpha\left(\frac{1}{2}\right)^{1/\beta}$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = -\frac{\beta\Gamma(\beta,-t\alpha)-\Gamma(1+\beta)}{(-t)^{\beta}\alpha^{\beta}} \qquad -\infty < t < \infty.$$

The characteristic function of X is mathematically intractable. The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{\alpha\beta}{1+\beta} \qquad V[X] = \frac{\alpha^2\beta}{(2+\beta)(1+\beta)^2}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{2(\beta-1)\sqrt{2+\beta}}{(3+\beta)\sqrt{\beta}}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3\left(2-\beta+3\beta^2\right)(2+\beta)}{\beta(3+\beta)(4+\beta)}.$$

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
X := [[x -> beta * x ^ (beta - 1) / alpha ^ beta], [0, alpha],
      ["Continuous", "PDF"]];
CDF(X);
SF(X);
HF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, kurtosis, and moment generating function.