Polya distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)

The shorthand $X \sim \text{Polya}(n, p, \beta)$ is used to indicate that the random variable X has the Polya distribution with parameters n, p, and β. A Polya random variable X with parameters n, p, and β has probability mass function

$$f(x) = \frac{\binom{n}{x} \prod_{j=0}^{x-1} (p + j\beta) \prod_{k=0}^{n-x-1} (1 - p + k\beta)}{\prod_{i=0}^{n-1} (1 + i\beta)} \qquad x = 0, 1, 2, \ldots, n,$$

for all $n = 1, 2, \ldots$, $0 < p < 1$, and $\beta > 0$.

The cumulative distribution, survivor function, hazard function, cumulative hazard function, and inverse distribution function, moment generating function, and characteristic function on the support of X are mathematically intractable.

The population mean of X is

$$E[X] = -\frac{\sin\left(\frac{\pi(-1+\beta+p)}{\beta}\right) p n \sin\left(\frac{\pi(n\beta+1)}{\beta}\right)}{\sin\left(\frac{\pi}{\beta}\right) \sin\left(\frac{\pi(n\beta-\beta-p+1)}{\beta}\right)}.$$

APPL verification: The APPL statements

```
X := [[x -> binomial(n, x) * product(p + j * beta, j = 0 .. x - 1) *
      product(1 - p + k * beta, k = 0 .. n - x - 1) /
      (product(1 + i * beta, i = 0 .. n - 1))],
      [0 .. n], ["Discrete", "PDF"]];
Mean(X);
Variance(X);
Skewness(X);
MGF(X);
```

return the population mean, variance, skewness, and moment generating function.