Noncentral chi-square distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim$ noncentral chisquare(δ, n) is used to indicate that the random variable X has the noncentral chi-square distribution with positive integer parameter n and nonnegative noncentrality parameter δ. A noncentral chi-square random variable X with parameters δ and n has probability density function

$$f(x) = \sum_{k=0}^{\infty} \frac{e^{\frac{-\delta-x}{2}} \left(\frac{\delta}{2}\right)^k x^{\frac{n+2k}{2}-1}}{\left(2^{\frac{n+2k}{2}}\right) \Gamma\left(\frac{n+2k}{2}\right) k!} \qquad x > 0.$$

The cumulative distribution, survivor, hazard, cumulative hazard, and inverse distribution on the support of X are mathematically intractable.

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \frac{e^{\delta t/(1-2t)}}{(1-2t)^{n/2}} \qquad 2t < 1.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = \frac{e^{\delta it/(1-2it)}}{(1-2it)^{n/2}} \qquad 2t < 1.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \delta + n \qquad V[X] = 2(n+2\delta) \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{2^{3/2}(n+3\delta)}{(n+2\delta)^{3/2}}$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = 3 + \frac{12(n+4\delta)}{(n+2\delta)^2}.$$

APPL verification: The APPL statements

```
assume(delta >= 0);
assume(n, posint);
X := [[x -> sum(exp(-delta / 2) * (delta / 2) ^ k / factorial(k) * exp(-x / 2) *
            x ^ (n / 2 + k - 1) / (2 ^ (n / 2 + k) * GAMMA(n / 2 + k)),
            k = 0 .. infinity)], [0, infinity], ["Continuous", "PDF"]];
MGF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the moment generating function, population mean, variance, skewness, and kurtosis.