

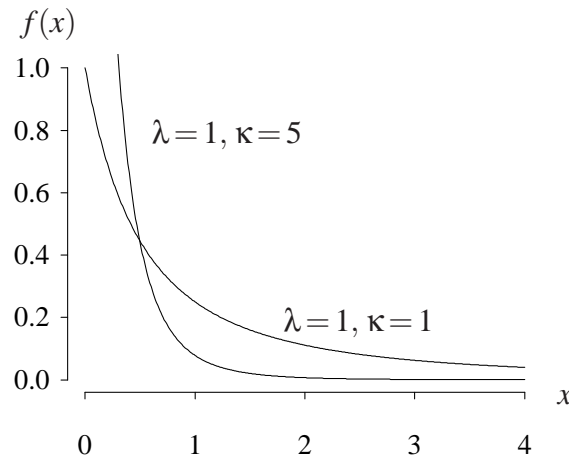
Lomax Distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim \text{Lomax}(\lambda, \kappa)$ is used to indicate that the random variable X has the Lomax distribution with parameters λ and κ . A Lomax random variable X with scale parameter λ and shape parameter κ has probability density function

$$f(x) = \frac{\lambda\kappa}{(1 + \lambda x)^{\kappa+1}} \quad x > 0,$$

for $\lambda > 0$ and $\kappa > 0$.

The probability density function with two different parameterizations is illustrated below:



Using the original parameterization, the cumulative distribution function on the support of X is

$$F(x) = P(X \leq x) = 1 - (1 + \lambda x)^{-\kappa} \quad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \geq x) = (1 + \lambda x)^{-\kappa} \quad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\lambda\kappa}{1 + \lambda x} \quad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \kappa \ln(1 + \lambda x) \quad x > 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = \frac{(1 - u)^{-1/\kappa} - 1}{\lambda} \quad 0 < u < 1.$$

The median of X is

$$\frac{2^{1/\kappa} - 1}{\lambda}.$$

The moment generating function of X is mathematically intractable. The population mean of X is

$$E[X] = \frac{\lambda}{\kappa - 1}$$

provided $\kappa > 1$. The population variance, skewness, and kurtosis of X are mathematically intractable.

For X_1, X_2, \dots, X_n mutually independent Lomax(λ) random variables, the maximum likelihood estimator for α is

$$\hat{\alpha} = \frac{n}{\sum_{i=1}^n \ln \left(1 + x_i / \hat{\lambda} \right)}.$$

An iteration procedure must be used to solve the following equation for $\hat{\lambda}$, and then substitute in the previous to obtain $\hat{\alpha}$.

$$\frac{n}{\hat{\lambda} \left(\sum_{i=1}^n \frac{x_i}{\hat{\lambda}^2 + \hat{\lambda} x_i} \right)} - 1 = \frac{n}{\sum_{i=1}^n \ln \left(1 + \frac{x_i}{\hat{\lambda}} \right)}$$