Theorem Random variates from the logistic-exponential distribution with parameters α and β can be generated in closed-form by inversion.

Proof The logistic-exponential(α, β) distribution has cumulative distribution function

$$F(x) = \frac{(e^{\alpha x} - 1)^{\beta}}{1 + (e^{\alpha x} - 1)^{\beta}} \qquad\qquad x > 0.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \frac{\ln[(u/(1-u))^{1/\beta} + 1]}{\alpha} \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the logistic-exponential(α, β) distribution is

generate $U \sim U(0, 1)$
$X \leftarrow \ln[(U/(1-U))^{1/\beta} + 1]/\alpha$
return(X)

APPL verification: The APPL statements

```
X := [[x -> (exp(alpha * x) - 1) ^ beta / (1 + (exp(alpha * x) - 1) ^ beta)],
      [0, infinity], ["Continuous", "CDF"]];
IDF(X);
```

verify the inverse distribution function of a logistic-exponential(α, β) random variable.