Theorem The error(a, b, c) distribution is a special case of the Laplace(α_1, α_2) distribution when $\alpha_1 = \alpha_2$.

Proof Let the random variable X have the Laplace(α_1, α_2) distribution with probability density function

$$f_X(x) = \begin{cases} \frac{1}{\alpha_1+\alpha_2}\, e^{x/\alpha_1} & x < 0 \\ \frac{1}{\alpha_1+\alpha_2}\, e^{-x/\alpha_2} & x > 0. \end{cases}$$

When $\alpha_1 = \alpha_2 = \alpha$ we have

$$\begin{aligned} f_X(x) &= \begin{cases} \frac{1}{\alpha_1+\alpha_2}\, e^{x/\alpha_1} & x < 0 \\ \frac{1}{\alpha_1+\alpha_2}\, e^{-x/\alpha_2} & x > 0 \end{cases} \\ &= \begin{cases} \frac{1}{2\alpha}\, e^{x/\alpha} & x < 0 \\ \frac{1}{2\alpha}\, e^{-x/\alpha} & x > 0, \end{cases} \end{aligned}$$

which is the probability density function of the error(a, b, c) random variable when $a = 0$, $b = \alpha/2$, and $c = 2$.

APPL verification: The APPL statements

```
alpha1 := alpha;
alpha2 := alpha;
X := [[x -> (1 / (alpha1 + alpha2)) * exp(x / alpha1),
            (1 / (alpha1 + alpha2)) * exp(-x / alpha2)],
      [-infinity, 0, infinity], ["Continuous", "PDF"]];
simplify(X[1][1](x));
simplify(X[1][2](x));
```

yield the probability density function of an error(a, b, c) random variable

$$f_Y(y) = \begin{cases} \frac{1}{2\alpha}\, e^{y/\alpha} & y < 0 \\ \frac{1}{2\alpha}\, e^{-y/\alpha} & y > 0 \end{cases}$$

when $a = 0$, $b = \alpha/2$, and $c = 2$.