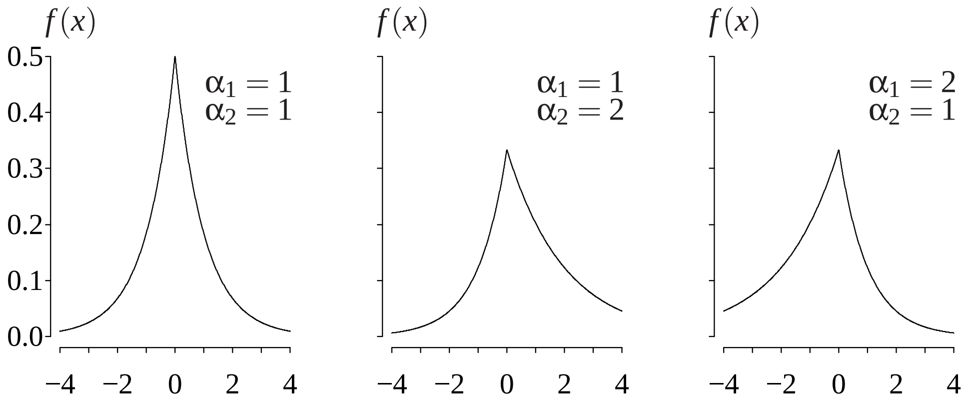
Laplace distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{Laplace}(\alpha_1, \alpha_2)$ is used to indicate that the random variable X has the Laplace distribution with positive scale parameters α_1 and α_2. A Laplace random variable X has probability density function

$$f(x) = \begin{cases} (1/(\alpha_1+\alpha_2))e^{x/\alpha_1} & x < 0 \\ (1/(\alpha_1+\alpha_2))e^{-x/\alpha_2} & x \ge 0 \end{cases}$$

for $\alpha_1, \alpha_2 > 0$. The Laplace distribution is an alternative to the normal distribution with heavier tails. The probability density function for three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \begin{cases} (\alpha_1/(\alpha_1+\alpha_2))e^{x/\alpha_1} & x < 0 \\ 1-(\alpha_2/(\alpha_1+\alpha_2))e^{-x/\alpha_2} & x \ge 0. \end{cases}$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \begin{cases} 1-(\alpha_1/(\alpha_1+\alpha_2))e^{x/\alpha_1} & x < 0 \\ (\alpha_2/(\alpha_1+\alpha_2))e^{-x/\alpha_2} & x \ge 0. \end{cases}$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \begin{cases} e^{x/\alpha_1}/(\alpha_1+\alpha_2+\alpha_1 e^{x/\alpha_1}) & x < 0 \\ 1/\alpha_2 & x \ge 0. \end{cases}$$

The inverse distribution function of X is

$$F^{-1}(u) = \begin{cases} -\alpha_1\left(-\ln(\alpha_1+\alpha_2)+\ln(\alpha_1)-\ln(u)\right) & 0 < u < \alpha_1/(\alpha_1+\alpha_2) \\ -\alpha_2\left(\ln(\alpha_1+\alpha_2)-\ln(\alpha_2)+\ln(1-u)\right) & \alpha_1/(\alpha_1+\alpha_2) \le u < 1. \end{cases}$$

The moment generating function and the characteristic function of X are mathematically intractable.

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \alpha_2 - \alpha_1 \qquad V[X] = \alpha_1^2 + \alpha_2^2$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = -\frac{2(\alpha_1^3-\alpha_2^3)}{(\alpha_1^2+\alpha_2^2)^{3/2}} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3(3\alpha_1^4+2\alpha_2^2\alpha_1^2+3\alpha_2^4)}{(\alpha_1^2+\alpha_2^2)^2}.$$

APPL verification: The APPL statements

```
assume(alpha1 > 0);
assume(alpha2 > 0);
X := [[x -> exp(x / alpha1)/(alpha1 + alpha2),
       x -> exp(-x / alpha2)/(alpha1 + alpha2) ],
       [-infinity, 0, infinity], ["Continuous", "PDF"]];
CDF(X);
SF(X);
HF(X);
IDF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution function, survivor function, hazard function, population mean, variance, skewness, and kurtosis.