Theorem [UNDER CONSTRUCTION!] The limiting distribution of the inverse Gaussian distribution as $\lambda \to \infty$ is the standard normal distribution.

Proof [UNDER CONSTRUCTION!] Let the random variable X have the inverse Gaussian(λ, μ) distribution with probability density function

$$f(x) = \sqrt{\frac{\lambda}{2\pi\, x^3}}\, e^{-\frac{\lambda(x-\mu)^2}{2x\mu^2}} \qquad x > 0.$$

The mean of this random variable is $E[X] = \mu$. The variance of this random variable is $V[X] = \mu^3/\lambda$. Divide the random variable X by its standard deviation prior to taking the limit. Consider the transformation $Y = g(X) = X/\sqrt{\mu^3/\lambda}$. This is a 1–1 transformation from $\mathcal{X} = \{x \mid x > 0\}$ to $\mathcal{Y} = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = Y\sqrt{\mu^3/\lambda}$ and Jacobian

$$\frac{dX}{dY} = \sqrt{\mu^3/\lambda}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= \sqrt{\frac{\lambda}{2\pi\,(y\sqrt{\mu^3/\lambda})^3}}\, e^{-\frac{\lambda((y\sqrt{\mu^3/\lambda})-\mu)^2}{2\left(y\sqrt{\mu^3/\lambda}\right)\mu^2}} \left|\sqrt{\mu^3/\lambda}\right| \qquad y > 0. \end{aligned}$$

Taking the limit as $\lambda \to \infty$ yields

$$\lim_{\lambda\to\infty} \sqrt{\frac{\lambda}{2\pi\,(y\sqrt{\mu^3/\lambda})^3}}\, e^{-\frac{\lambda((y\sqrt{\mu^3/\lambda})-\mu)^2}{2\left(y\sqrt{\mu^3/\lambda}\right)\mu^2}} \left|\sqrt{\mu^3/\lambda}\right|.$$

The result appears on page 121 of Forbes, Evans, Hastings, and Peacock (2011), *Statistical Distributions*, Fourth Edition, John Wiley and Sons.