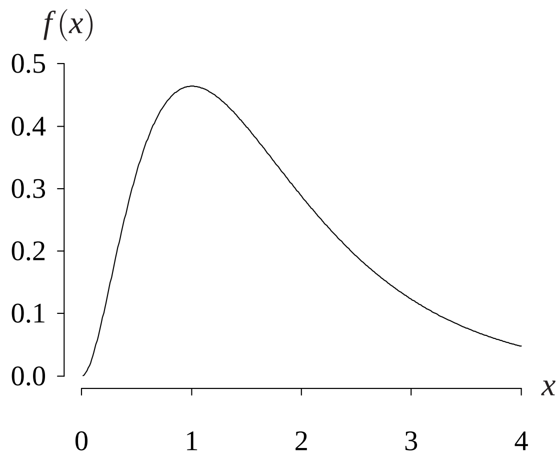
Hypoexponential distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{hypoexponential}(\vec{\alpha})$ is used to indicate that the random variable X has the hypoexponential distribution with positive vector parameter $\vec{\alpha}$ such that $\alpha_i \neq \alpha_j$ for $i \neq j$. If $T_i \sim \text{exponential}(\alpha_i)$ for $i = 1, 2, \ldots, n$, and $T_1, T_2, \ldots, T_n$ are mutually independent random variables, then $T = T_1 + T_2 + \cdots + T_n$ has the hypoexponential distribution. A hypoexponential random variable X has probability density function

$$f(x) = \sum_{i=1}^{n} (1/\alpha_i) e^{-x/\alpha_i} \left(\prod_{j=1,\, j \neq i}^{n} \frac{\alpha_i}{\alpha_i - \alpha_j} \right)$$

for $x > 0$. The probability density function for $\vec{\alpha} = (0.25, 0.5, 1)$ is illustrated below.



For the case where $n = 2$, the cumulative distribution function on the support of X is

$$F(x) = P(X \leq x) = 1 - \frac{1}{\alpha_1 - \alpha_2} \left(\alpha_2 e^{-x/\alpha_2} - \alpha_1 e^{-x/\alpha_1} \right) \qquad x > 0.$$

The general cumulative distribution, survivor, hazard, cumulative hazard, moment generating, and characteristic functions on the support of X are mathematically intractable.

The population mean, variance, and skewness of X are

$$E[X] = \sum_{i=1}^{n} \alpha_i \qquad V[X] = \sum_{i=1}^{n} \alpha_i^2 \qquad E\left[\left(\frac{X - \mu}{\sigma} \right)^3 \right] = \frac{2 \sum_{i=1}^{n} \alpha_i^3}{\left(\sum_{i=1}^{n} \alpha_i^2 \right)^{3/2}}.$$

APPL verification: The APPL statements

```
X:= HypoExponentialRV([1 / alpha1, 1 / alpha2, 1 / alpha3]);
Mean(X);
Variance(X);
Skewness(X);
```

verify the population mean, variance, and skewness for the special case of $n = 3$.