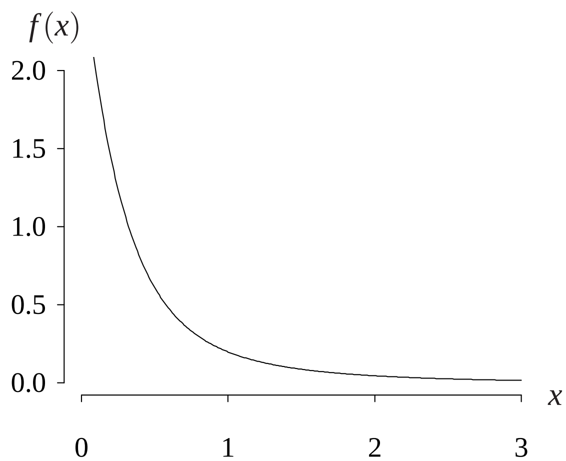
Hyperexponential distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{hyperexponential}(\vec{\alpha}, \vec{p})$ is used to indicate that the random variable X has the hyperexponential distribution with parameters $\vec{\alpha}$ and $\vec{p}$. A hyperexponential random variable X with parameters $\vec{\alpha}$ and $\vec{p}$ has probability density function

$$f(x) = \sum_{i=1}^{n} \frac{p_i}{\alpha_i} e^{-x/\alpha_i} \qquad x > 0$$

for all $\alpha_i, p_i > 0$ such that $\sum_{i=1}^{n} p_i = 1$. The probability density function for $\vec{\alpha} = (0.25, 0.5, 1)$ and $\vec{p} = (0.5, 0.25, 0.25)$ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = 1 - \sum_{i=1}^{n} p_i e^{-x/\alpha_i} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \le x) = \sum_{i=1}^{n} p_i e^{-x/\alpha_i} \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S((x)} = \frac{\sum_{i=1}^{n} \frac{p_i}{\alpha_i} e^{-x/\alpha_i}}{\sum_{i=1}^{n} p_i e^{-x/\alpha_i}} \qquad x > 0.$$

The inverse distribution function and characteristic function are both mathematically intractable.

The moment generating function over the support of X is

$$M(t) = E[e^{tX}] = \sum_{i=1}^{n} \frac{p_i}{1 - \alpha_i t} \qquad |t| < \frac{1}{\min_j \alpha_j}.$$

The population mean of X is

$$E[X] = \sum_{i=1}^{n} p_i \alpha_i.$$

The population skewness and kurtosis of X are all mathematically intractable.

APPL verification: The APPL statements

```
X := HyperExponentialRV([0.1, 0.3, 0.6], [alpha1, alpha2, alpha3]);
CDF(X);
SF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the population mean, variance, skewness, kurtosis, and moment generating function for the special case of $n = 3$ and $\vec{p} = (0.1, 0.3, 0.6)$.