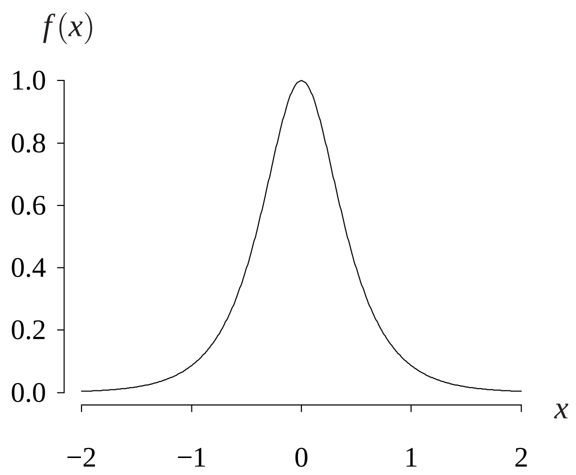
Hyperbolic-secant distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The hyperbolic-secant distribution has probability density function

$$f(x) = \operatorname{sech}(\pi x), \qquad -\infty < x < \infty,$$

where the hyperbolic-secant function is defined by

$$\operatorname{sech}(z) = \frac{2}{e^z + e^{-z}}$$

for $-\infty < z < \infty$. The probability density function is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{\pi + 2\arctan(\sinh(\pi x))}{2\pi} \qquad -\infty < x < \infty.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{\pi - 2\arctan(\sinh(\pi x))}{2\pi} \qquad -\infty < x < \infty.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = -\frac{2\pi}{\cosh(\pi x)(\pi - 2\arctan(\sinh(\pi x)))} \qquad -\infty < x < \infty.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \ln(2) + \ln(\pi) - \ln(\pi - 2\arctan(\sinh(\pi x))) \qquad -\infty < x < \infty.$$

The inverse distribution function of X is

$$F^{-1}(u) = \frac{\arcsin(\cot(\pi u))}{\pi} \qquad 0 < u < 1.$$

The median of X is 0.

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \int_{-\infty}^{\infty} \operatorname{sech}(\pi x) \qquad\qquad t > 0.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = \int_{-\infty}^{\infty} \operatorname{sech}(\pi x) \qquad\qquad t > 0.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = 0 \qquad\qquad V[X] = 1 \qquad\qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \qquad\qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = 2.$$

APPL verification: The APPL statements

```
X := HyperbolicSecantRV();
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
Mean(X);
Skewness(X);
MGF(X);
```

verify the cumulative distribution function, survivor function,hazard function, cumulative hazard function, inverse function, population mean, skewness, and moment generating function. APPL fails to verify the population variance and kurtosis.