Theorem The geometric distribution has the memoryless (forgetfulness) property.

Proof A geometric random variable X has the memoryless property if for all nonnegative integers s and t,

$$P(X \geq s + t \mid X \geq t) = P(X \geq s)$$

or, equivalently

$$P(X \geq s + t) = P(X \geq s)P(X \geq t).$$

The probability mass function for a geometric random variable X is

$$f(x) = p(1 - p)^x \qquad x = 0, 1, 2, \ldots .$$

The probability that X is greater than or equal to x is

$$P(X \geq x) = (1 - p)^x \qquad x = 0, 1, 2, \ldots .$$

So the conditional probability of interest is

$$\begin{aligned} P(X \geq s + t \,|\, X \geq t) &= \frac{P(X \geq s + t,\, X \geq t)}{P(X \geq t)} \\ &= \frac{P(X \geq s + t)}{P(X \geq t)} \\ &= \frac{(1 - p)^{s+t}}{(1 - p)^t} \\ &= (1 - p)^s \\ &= P(X \geq s), \end{aligned}$$

which proves the memoryless property.

APPL verification: The APPL statements

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simplify((1 - op(CDF(GeometricRV(p)))(s)[1]) * (1 - op(CDF(GeometricRV(p))(t)[1])));
1 - simplify(op(CDF(GeometricRV(p))(s + t)[1]));
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both yield the expression

$$(1 - p\ )^{s+t}.$$