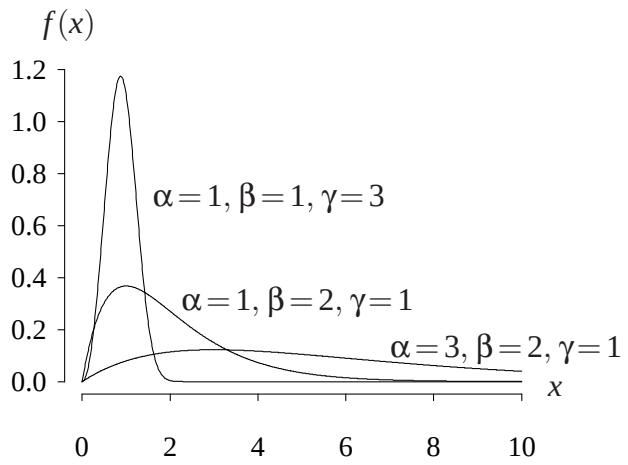
Generalized gamma distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html) The shorthand $X \sim$ generalized gamma(α, β, γ) is used to indicate that the random variable X has the generalized gamma distribution with real positive parameters α, β, and γ. A generalized gamma random variable X with scale parameter α, and shape parameters β, and γ has probability density function

$$f(x) = \frac{\gamma x^{\gamma\beta-1} e^{-(x/\alpha)^{\gamma}}}{\alpha^{\gamma\beta}\Gamma(\beta)} \qquad x > 0.$$

The probability density function with three different parameter combinations is illustrated below.



The cumulative distribution, survivor, hazard, cumulative hazard, inverse distribution, moment generating, and characteristic functions on the support of X are mathematically intractable. The population mean and variance of X are

$$E[X] = \frac{\alpha\Gamma(\beta+1/\gamma)}{\Gamma(\beta)} \qquad V[X] = \alpha^2 \left(\frac{\Gamma(\beta+2/\gamma)}{\Gamma(\beta)} - \left(\frac{\Gamma(\beta+1/\gamma)}{\Gamma(\beta)} \right)^2 \right).$$

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
assume(y > 0);
X := [[x -> y * x ^ (y * beta - 1) * exp(-(x / alpha) ^ y) /
       (alpha ^ (y * beta) * GAMMA(beta))], [0, infinity],
      ["Continuous", "PDF"]];
Mean(X);
Variance(X);
```

verify the population mean and variance. APPL also calculates the skewness and kurtosis, but the expressions are long.