Theorem The Pascal distribution is a special case of the gamma–Poisson distribution when $\alpha = (1-p)/p$ and $\beta = n$.

Proof The gamma–Poisson distribution has probability mass function

$$f(x) = \frac{\Gamma(\beta + x)\alpha^x}{\Gamma(\beta)(1+\alpha)^{\beta+x}x!} \qquad x = 0, 1, 2, \ldots .$$

When $\beta = n$ and $\alpha = (1-p)/p$, this reduces to

$$\begin{aligned} f(x) &= \frac{\Gamma(n+x)((1-p)/p)^x}{\Gamma(n)(1+(1-p)/p)^{n+x}x!} \\ &= \frac{(n+x-1)!(1-p)^x p^{n+x}}{(n-1)!p^x x!} \\ &= \binom{n+x-1}{x} p^n (1-p)^x \qquad x = 0, 1, 2, \ldots, \end{aligned}$$

which is the probability mass function of the Pascal distribution.

Maple verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
X := [[x -> GAMMA(beta + x) * alpha ^ x / (GAMMA(beta) * (1 + alpha)
            ^ (beta + x) * x!)], [0 .. infinity], ["Discrete", "PDF"]];
assume(p > 0);
additionally(p < 1);
assume(n, posint);
subs({alpha = (1 - p) / p, beta = n}, X[1][1](x));
```

yield the probability mass function of the Pascal distribution as parameterized in the proof.