Theorem The geometric distribution is a special case of the gamma–Poisson distribution with $\alpha = \beta = 1$ in the gamma–Poisson distribution and $p = \frac{1}{2}$ in the geometric distribution.

Proof A gamma–Poisson random variable X has probability mass function

$$f(x) = \frac{\Gamma(x+\beta)\alpha^x}{\Gamma(\beta)(1+\alpha)^{x+\beta}x!} \qquad x = 0, 1, 2, \ldots .$$

Substituting $\alpha = \beta = 1$ yields

$$f(x) = \frac{\Gamma(x+1)1^x}{\Gamma(1)(1+1)^{x+1}x!} = \frac{1}{2}\left(\frac{1}{2}\right)^x \qquad x = 0, 1, 2, \ldots ,$$

which is the probability mass function of a geometric random variable with $p = \frac{1}{2}$.

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
X := [[x -> GAMMA(x + beta) * alpha ^ x / (GAMMA(beta) * (1 + alpha) ^ (x + beta)
            * x!)], [0, infinity], ["Discrete", "PDF"]];
simplify(subs({alpha = 1, beta = 1}, X[1][1](x)));
```

yield the probability mass functions for a geometric random variable with parameter $p = 1/2$ whose support begins at 0.