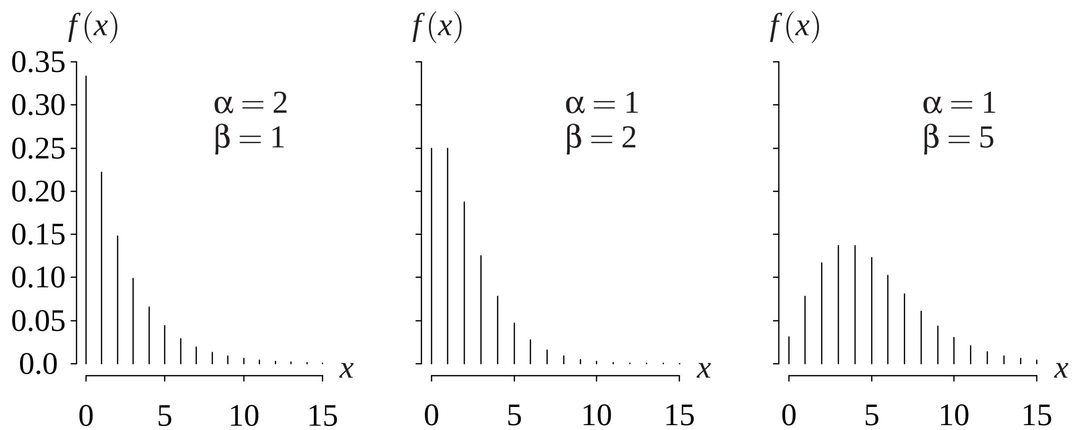
Gamma–Poisson distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim$ gamma–Poisson(α, β) is used to indicate that the random variable X has the gamma–Poisson distribution with positive parameters α and β. A gamma–Poisson random variable X has probability mass function

$$f(x) = \frac{\Gamma(x+\beta)\alpha^x}{\Gamma(\beta)(1+\alpha)^{\beta+x}x!} \qquad x = 0, 1, 2, \ldots$$

for any $\alpha, \beta > 0$. A gamma–Poisson random variable is a Poisson random variable with a random parameter μ which has the gamma distribution with parameters α and β. The probability mass function for three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = \sum_{w=0}^{x} \frac{\Gamma(w+\beta)\alpha^w}{\Gamma(\beta)(1+\alpha)^{\beta+w}w!} \qquad x = 0, 1, 2, \ldots$$

The moment generating function of X is

$$M(t) = E\left[e^{tx}\right] = \left(1+\alpha-\alpha e^t\right)^{-\beta} \qquad -\infty < t < \infty.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itx}\right] = \left(1+\alpha-\alpha e^{it}\right)^{-\beta} \qquad -\infty < t < \infty.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \alpha\beta \qquad V[X] = \alpha\beta + \alpha^2\beta$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{1+2\alpha}{\sqrt{\alpha\beta(1+\alpha)}} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3\alpha^2\beta+6\alpha^2+3\alpha\beta+6\alpha+1}{\alpha\beta(1+\alpha)}.$$