Theorem Exponential random variates can be generated in closed form by inversion.

Proof The exponential distribution has probability density function

$$f(x) = \frac{1}{\alpha} e^{-x/\alpha} \qquad\qquad x > 0$$

and cumulative distribution function

$$F(x) = 1 - e^{-x/\alpha} \qquad\qquad x > 0.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = -\alpha \ln(1 - u) \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the exponential distribution is

generate $U \sim U(0, 1)$
$X \leftarrow -\alpha \ln(1 - u)$
return(X)

APPL verification:

```
assume(alpha > 0);
X := ExponentialRV(1 / alpha);
CDF(X);
IDF(X);
simplify(%[1][1](u));
```

which yields the same form for the cumulative distribution function and IDF as given above.