Theorem The exponential distribution has the memoryless (forgetfulness) property.

Proof A variable X with positive support is memoryless if for all $t > 0$ and $s > 0$

$$P(X > s + t \mid X > t) = P(X > s)$$

or, using the definition of conditional probability,

$$P(X > s + t) = P(X > s)P(X > t).$$

An exponential random variable with population mean α has survivor function

$$P(X \geq x) = e^{-x/\alpha} \qquad\qquad x > 0.$$

Thus, we have

$$\begin{aligned} P(X > s)P(X > t) &= e^{-s/\alpha}e^{-t/\alpha} \\ &= e^{-(s+t)/\alpha} \\ &= P(X > s + t). \end{aligned}$$

So the exponential distribution has the memoryless property.

APPL verification: The APPL statements given below confirm the memoryless property for the exponential distribution.

```
assume(alpha > 0);
X := [[x -> exp(-x / alpha) / alpha], [0, infinity],
      ["Continuous", "PDF"]];
simplify(op(SF(X)(s)[1]) * op(SF(X)(t)[1]));
simplify(op(SF(X)(s + t)[1]));
```

The last two statements both yield

$$e^{-(s+t)/\alpha}.$$